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Stationary particles in the field of magnetized slowly rotating neutron stars

Pavel Bakala[a], Martin Urbanec, Eva Šrámková,
Zdeněk Stuchlík and Gabriel Török

Institute of Physics, Faculty of Philosophy & Science, Silesian University in Opava,
Bezručovo nám. 13, CZ-746 01 Opava, Czech Republic
[a]pavel.bakala@fpf.slu.cz

ABSTRACT
We study circular motion of charged test particles in the field of magnetized slowly rotating neutron stars. The gravitational field is approximated by the Lense–Thirring geometry, the magnetic field is of the standard dipole character. Using a fully-relativistic approach we determine influence of the electromagnetic interaction (both attractive and repulsive) on the circular motion. We focus on the behaviour of the orbital frequency of the motion. Components of the four-velocity of the orbiting charged test particles are obtained by numerical solution of equations of motion. The role of the combined effect of the neutron star magnetic field and its rotation in the character of the orbital frequency is discussed. It is demonstrated that even in the Lense–Thirring spacetime particles being static relative to distant observers can exist due to the combined gravo-electromagnetic interaction.

Keywords: Lense–Thirring – neutron star – magnetic and electric fields – accretion

1 INTRODUCTION

Charged particle motion in strong gravitational and electromagnetic fields of black holes and neutron stars enables us to understand the nature of combined effects of these fields and their role in astrophysical phenomena. The motion has been investigated both for Kerr–Newman black holes having intrinsically coupled gravitational and electromagnetic fields and for strong gravitating objects (black holes and neutron stars) with a test electromagnetic field influenced by gravity (see, e.g. Johnston and Ruffini, 1974; Prasanna and Vishveshwara, 1978; Prasanna, 1980; Calvani et al., 1982; Bálek et al., 1989; Bičák et al., 1989; Stuchlík and Hledík, 1998; Stuchlík et al., 1999; Abdujabbarov and Ahmedov, 2009; Frolov and Shoom, 2010). Motion of charged particles in the magnetic field generated by accretion discs orbiting black holes was discussed in (Znajek, 1976; Mobarry and Lovelace, 1986). The magnetic field tied to a neutron star could substantially influence the structure of an equatorial accretion disc orbiting the neutron star and has been studied in (Kluźniak and Rappaport, 2007).

In the case of motion in test fields on strong gravity backgrounds, the equations of motion are complex and have to be integrated numerically (Prasanna and Vishveshwara, 1978; Prasanna and Sengupta, 1994; Preti, 2004). Numerical integration of the motion equations

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gives a number of interesting results, but is not sufficient for a complete classification and understanding of the motion in the equatorial plane. In order to extend the understanding of the charged particle motion, the quasi-circular equatorial epicyclic motion corresponding to oscillations of particles around stable circular orbits has been studied (Bakala et al., 2010). Such epicyclic motion can be excited in the innermost parts of the accretion discs orbiting a neutron star by inhomogeneities (mountains) on its surface (Stuchlík et al., 2008). Recently, off-equatorial circular orbits were discussed in astrophysically relevant situations (Kovář et al., 2008, 2010; Kopáček et al., 2010). Of high interest is the equatorial motion, especially the circular and quasi-circular orbits of charged test particles that seem to be crucial from the point of view of accretion processes. Moreover, quite recently, fluid charged tori were discussed in the approximation of zero conductivity (Kovář et al., 2011); such dielectric tori could be also relevant in some astrophysically interesting situations.

Here we focus attention on the equatorial orbital motion in the combined gravitational and dipole magnetic fields related to a slowly rotating neutron star. We generalize our previous results obtained under much simpler case of neutron star represented by the Schwarzschild geometry and the related magnetic field (Bakala et al., 2010). We assume a dipole field whose axis of symmetry coincides with the axis of neutron star's rotation. The spacetime outside the neutron star is described by the Lense–Thirring geometry that reflects the slow rotation of the neutron star and influences the structure of the magnetic field – the effects of frame-dragging are thus considered in the linear approximation. Such approximation is suitable for describing the charged particles motion around slowly rotating neutron stars with a relatively weak magnetic field which does not affect the spacetime curvature in the vicinity of the neutron star, but its structure is governed by the neutron star spacetime structure.[1]

In our study we focus our attention on the possibility of existence of charged particles that appear stationary to distant observers. Existence of such particles was demonstrated for ultrarelativistic charged particles located near the black hole horizon of charged and rotating (Kerr–Newman) black hole (Bálek et al., 1989). Here we test such possibility in different physical conditions when the interplay of gravitational dragging and electromagnetic force can imply interesting and unexpected results. The problem of the epicyclic motion and the related frequencies (see e.g. Aliev and Galtsov, 1981; Abramowicz and Kluźniak, 2005; Török and Stuchlík, 2005) is postponed for future studies.

2 LENSE–THIRRING GEOMETRY AND DIPOLE MAGNETIC FIELD OF SLOWLY ROTATING NEUTRON STARS

The external gravitational field of slowly rotating neutron or strange stars is sometimes approximated by the Lense–Thirring metric[2] (Lense and Thirring, 1918; Hartle and Sharp,

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[1] The neutron star magnetic field is however fully dominant over the magnetic field generated by the currents in the disc.

[2] The term "Lense–Thirring metric" is substituted frequently by the term "slow-rotation approximation" (see Konno and Kojima (2000)).

1967; Hartle, 1967), with line element given by

$$\mathrm{d}s^2 = -\eta(r)^2\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\eta(r)^2} + r^2\Big[\mathrm{d}\theta^2 + \sin^2\theta\,\Big(\mathrm{d}\phi^2 - 2\omega\,(r)\,\mathrm{d}t\,\mathrm{d}\phi\Big)\Big], \tag{1}$$

where the function $\eta(r)$ reads

$$\eta(r) \equiv \left(1 - \frac{2M}{r}\right)^{1/2}. \tag{2}$$

The Lense–Thirring angular velocity $\omega(r)$ can be interpreted as angular velocity of freely falling observers relative to static observers at infinity and outside the neutron star is given by

$$\omega(r) = \frac{2J}{r^3}, \tag{3}$$

where J is the total angular momentum of the neutron star with mass M and radius R. Using the moment of inertia $I(M, R)$ and angular velocity of the (rigidly) rotating star Ω_{star} measured by a static observer at infinity, we can write $J = I(M, R)\Omega_{\mathrm{star}}$. The rotational parameter of the neutron star (called spin) is given by $a = J/M^2$. We have adopted here geometric units, $c = G = 1$, that we will use throughout the paper.

In Rezzolla et al. (2001), an analytical solution of the Maxwell equations is presented for a general orientation of the dipole magnetic field in the Lense–Thirring metric (1) to first order in J, including the conditions for matching the internal spacetime of the star under assumption of both infinite and finite conductivity of the star interior. We assume for simplicity the symmetry axis of the magnetic dipole identical with rotation axis (zero declination) and infinitely conductive star interior implying force lines frozen into the star and dragged by its rotation. Under such assumptions, the relatively complex general dipole solution is reduced to much simpler form (Konno and Kojima, 2000), with the azimuthal component of the electromagnetic 4-potential A_ϕ being identical with the Schwarzschildian case (e.g. Wasserman and Shapiro, 1983; Braje and Romani, 2001)

$$A_\phi = -f(r)\frac{\mu\sin^2\theta}{r}, \tag{4}$$

i.e. to the magnetic dipole solution of the Maxwell equations in the flat spacetime corrected by the general relativistic factor $f(r, M)$ that is given by

$$f(r) = \frac{3r^3}{8M^3}\left[\ln\eta(r)^2 + \frac{2M}{r}\left(1 + \frac{M}{r}\right)\right]. \tag{5}$$

In contrast to the dipole solution in the static spherically symmetric spacetime, the 4-potential contains also non-zero electrical (time) component that can be expressed in the form

$$A_t(r,\theta) = a_{t0}(r) + a_{t2}(r)\mathrm{P}_2(\cos\theta)\;; \tag{6}$$

P_2 is the Legendre polynome of the 2nd kind (Konno and Kojima, 2000). The terms a_{t0} a a_{t2} are given by the Maxwell equations and take the form

$$a_{t0} = \frac{c_0}{r} + \frac{J\mu}{2M^3r^2}(3r - M) + \frac{J\mu}{4M^4r}(3r - 4M)\ln\eta^2(r)\,, \tag{7}$$

$$\begin{aligned} a_{t2} = {} & \frac{c_1}{M^2}(r - M)(r - 2M) \\ & + c_2\left[\frac{2}{Mr}\left(3r^2 - 6Mr + M^2\right) + \frac{3}{M^2}\left(r^2 - 3Mr + 2M^2\right)\ln\eta^2(r)\right] \\ & - \frac{J\mu}{2M^6r^2}\bigg[\left(9r^4 - 3Mr^3 - 30M^2r^2 + 8M^3r + 2M^4\right) \\ & + \left(12r^4 - 36Mr^3 + 24M^2r^2 + M^3r\right)\ln\eta^2(r)\bigg]\,, \end{aligned} \tag{8}$$

where c_0, c_1 a c_2 are integration constants (Konno and Kojima, 2000). The first constant c_0 corresponds to the electric charge of the star and it is astrophysically natural to put ($c_0 = 0$). Requirement of regularity of the solution at infinity implies (Konno and Kojima, 2000)

$$c_1 = \frac{9J\mu}{2M^4}\,; \tag{9}$$

c_2 can be fixed by the matching conditions on the star surface. Assuming perfectly conducting interior of a star rotating with angular momentum Ω_{star} and frozen-in magnetic field ($u^\mu F_{\mu\nu} = 0$, $u^\mu = (u^t, 0, 0, \Omega_{\mathrm{star}}u^t)$), we arrive at (Konno and Kojima, 2000)

$$\begin{aligned} c_2 = {} & \bigg\{\frac{\mu J}{M^5R^2}\left(12R^3 - 24MR^2 + 4M^2R + M^3\right) \\ & + \frac{\mu J}{2M^6R}\left(12R^3 - 36MR^2 + 24M^2R + M^3\right)\ln\eta^2(r) \\ & - \frac{\mu\Omega_{\mathrm{star}}}{4M^3}\left[2MR + 2M^2 + R^2\ln\eta^2(r)\right]\bigg\} \\ & \bigg/\left[\frac{2}{MR}\left(3R^2 - 6MR + M^2\right) + \frac{3}{M^2}\left(R^2 - 3MR + 2M^2\right)\ln\eta^2(r)\right]. \end{aligned} \tag{10}$$

The Maxwell tensor $F_{\mu\nu}$ related to the four-potential A_μ by

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}\,, \tag{11}$$

has four independent non-vanishing components

$$F_{r\phi} = \frac{\mu\sin^2\theta\big(f(r) - rf'(r)\big)}{r^2}\,, \tag{12}$$

$$F_{\theta\phi} = -\frac{\mu f(r)\sin 2\theta}{r}\,, \tag{13}$$

$$F_{tr} = -a'_{t0}(r) - \frac{a'_{t2}(r)}{4}(1 + 3\cos 2\theta)\,, \tag{14}$$

and

$$F_{t\theta} = a_{t2}(r)\, 3\cos\theta \sin\theta\,. \tag{15}$$

corresponding to appropriate parts of electric and magnetic field three-vectors in the frames of local observers. Note that "coma" in Eqs. (12, 14) denotes partial derivative with respect to the radial coordinate r.

Notice that the electric component of the 4-potential is in astrophysically relevant case of electrically uncharged star induced only by the star rotation and the effect of dragging of inertial frames is indicated by its dependence on the angular velocity of the star $\Omega_{\rm star}$ and its internal angular momentum J.

2.1 Relation between spin and angular frequency

The internal angular momentum J and the angular velocity of the star $\Omega_{\rm star}$ are linearly connected by the moment of inertia through the relation $J = I\Omega_{\rm star}$. To find the value of angular velocity necessary for matching the condition given by Eq. (10) in terms of a dimensionless spin $a = J/M^2$, we can use the findings of Lo and Lin (2011) that the maximal value of spin, $a_{\rm max} = 0.7$, is almost the same for all masses and equations of state. Using a model of neutron star with mass $M = 1.5\,M_\odot$ we can find (see, e.g. Haensel et al., 2009) that maximal frequency $\nu^{\rm max}_{\rm star} = \Omega^{\rm max}_{\rm star}/2\pi$ for neutron star is roughly 750–1200 Hz. The exact value of the maximal frequency depends very significantly on the assumed equation of state of the neutron star matter (Lattimer and Prakash, 2001; Říkovská Stone et al., 2003; Urbanec et al., 2010). Since we are dealing here with a neutron star test model, in further analysis we use the value of $\nu^{\rm max}_{\rm star} = 1000$ Hz. Therefore, the linear relation between the spin a and the rotational frequency $\Omega_{\rm star}$ can be written in the form

$$\Omega_{\rm star} = \alpha a\,, \tag{16}$$

where parameter α is given as the ratio of maximal values of neutron star's spin and the angular velocity;

$$\alpha = \Omega^{\rm max}_{\rm star}/a_{\rm max}\,. \tag{17}$$

2.2 Intrinsic magnetic dipole moment

Intrinsic magnetic dipole moment of a neutron star μ can be obtained from the presumed magnetic field strength at the neutron star surface. Locally measured magnetic field strength is given by the projection of the Maxwell tensor into the orthonormal basis of a observer connected with the surface of the star, $F_{\hat\alpha\hat\beta}{=}e_{\hat\alpha}{}^{\mu}e_{\hat\beta}{}^{\nu}F_{\mu\nu}$. The tetrad related to the observers at the surface of the neutron star is given by the relations

$$e_{\hat t} = \left\{u^t, 0, 0, \Omega_{\rm star}\, u^t\right\}, \qquad e_{\hat r} = \left\{0, \eta(r), 0, 0\right\}, \tag{18}$$

$$e_{\hat{\theta}} = \left\{0, 0, \frac{1}{r}, 0\right\}, \qquad e_{\hat{\phi}} = \left\{0, 0, 0, \frac{1}{r\,\sin\theta}\right\},$$

The magnetic components of the Maxwell tensor of the electromagnetic field in the Lense–Thirring metric correspond to the static (Schwarzschild) solution – therefore, the relation of the magnetic dipole moment of the neutron star and the magnetic induction on its surface takes precisely the same form as in (Bakala et al., 2010)

$$\mu = \frac{4M^3R^{3/2}\sqrt{R-2M}}{6M(R-M)+3R(R-2M)\log\eta\,(R)^2}B^{\hat{\theta}}\,. \tag{19}$$

We use here as the test model for our analysis a neutron star with a rather weak magnetic field strength, $B = 10^7$ Gauss $\simeq 2.875 \times 10^{-16}\,\mathrm{m}^{-1}$,[3] mass $M = 1.5M_{\odot} \simeq 2216.85$ m and radius $R = 4M \simeq 8867.4$ m, as in our previous analysis of the static geometry (Bakala et al., 2010). Then we have $\mu = 1.06 \times 10^{-4}\,\mathrm{m}^2 = 2.157 \times 10^{-11}M^2$.

3 EQUATORIAL CIRCULAR MOTION

In a curved spacetime with presence of an electromagnetic field, the Lorentz equation of motion for a charged test particle of mass m and charge q reads

$$\frac{\mathrm{d}U^{\mu}}{\mathrm{d}\tau} + \Gamma^{\mu}_{\alpha\beta}U^{\alpha}U^{\beta} = \tilde{q}\,F^{\mu}_{\nu}U^{\nu}\,, \tag{20}$$

where U^{μ} is the four-velocity and $\tilde{q} \equiv q/m$ is the specific charge of the particle.

Symmetry properties of the spacetime geometry (1) and electromagnetic field (4) allow for charged test particles circular motion restricted to the equatorial plane $\theta = \pi/2$. The four-velocity then has only two non-vanishing components, $U^{\mu} = (U^t, 0, 0, U^{\phi})$. Solving the radial component of Eq. (20) together with the normalization condition $U^{\mu}U_{\mu} = -1$ for metric (1) and potential (4) we obtain two pair of implicit equations for nonzero components of U^{μ} in the form

$$U^t_{\pm} = \left(\pm\sqrt{4a^2M^4(U^{\phi})^2 + (r-2M)r\left(1+(U^{\phi})^2r^2\right)} - 2aM^2U^{\phi}\right)\Big/\left(r-2M\right), \tag{21}$$

$$U^{\phi}_{\pm} = \left(\frac{1}{2}r^{-3}\right)\Big[-2aM^2U^t - \tilde{q}\mu\Phi(r) \pm\sqrt{\left(2aM^2U^t + \tilde{q}\mu\Phi(r)\right)^2 + 2r^3U^t\left(2MU^t + \tilde{q}r^2\Sigma(r)\right)}\Big]. \tag{22}$$

Here and hereafter $\Phi(r)$ and $\Sigma(r)$ are given by

$$\Phi(r) \equiv f(r) - rf'(r)\,, \tag{23}$$

and

$$\Sigma(r) \equiv a'_{t0}(r) - 2a'_{t2}(r)\,. \tag{24}$$

[3] $B\,[\mathrm{cm}^{-1}] = (\mathrm{G}^{1/2}/c^2)\,B\,[\mathrm{Gauss}] \simeq 2,875 \times 10^{-25}\,B\,[\mathrm{Gauss}]$

For uncharged particles we arrive at the equations governing circular geodesic in the Lense–Thirring spacetime, where non-zero components of 4-velocity and orbital angular velocity read

$$U^{\phi}_{0\pm} = \pm\left[\frac{r^2}{M}(r-3M) + 2aM\left(aM \pm \sqrt{a^2M^2 + r^3/M}\right)\right]^{-1/2}, \tag{25}$$

$$U^{t}_{0\pm} = \left(aM \pm \sqrt{a^2M^2 + r^3/M}\right)U^{\phi}_{0\pm}, \tag{26}$$

$$\Omega_{0\pm} = \left(aM \pm \sqrt{a^2M^2 + r^3/M}\right)^{-1}. \tag{27}$$

In order to obtain appropriate angular velocities in the presence of the Lorentz force, the pair of Eqs. (21, 22) has to be solved numerically, taking into account only the physically relevant forward-directed time component of the 4-velocity U^t_+. The solution $\Omega_+ = U^{\phi}_+/U^t_+$ then corresponds in the geodesic limit to the corotating orbits and will be referred as corotating in the following, while the solution $\Omega_- = U^{\phi}_-/U^t_+$ will be referred as counterrotating (retrograde). Nevertheless, due to the electromagnetic interaction, in the case of the retrograde solution the real orientation of the orbital velocity depends on the values of the neutron star spin and the specific charge of the test particle.

For circular motion in the equatorial plane, the Lorentz force on the Rhs of the equations of motion (20) has the only non-zero, radial component that is given by the expression $K^r = \tilde{q}(F^r{}_t U^t + F^r{}_{\phi} U^{\phi})$, where the first term corresponds to the electric (coulombic) part of the interaction of the test charged particle with with electric field of the star induced by its spin, while the second term corresponds to the magnetic part of the interaction induced by the orbital motion of the charged particle. While orientation of the magnetic component depends both on the sign of the specific charge of the particle $\tilde{q}$ and the orientation of the orbital angular velocity $\Omega_{\pm}$, the electric component is for a fixed neutron star spin $a > 0$ always repulsive for $\tilde{q} > 0$, but attractive for $\tilde{q} < 0$. Nevertheless, both parts depend on the product of μ and $\tilde{q}$ determining magnitude of the whole electromagnetic interaction. Therefore, instead of changing magnitude and orientation of μ we can, without any loss of generality, study only influence of changes of the specific charge $\tilde{q}$ similarly as in analysis of the static Schwarzschild case (Bakala et al., 2010). However, we have to analyse separately the corotating and retrograde orbits due to the rotation of the neutron star. We analyse behaviour of orbiting test particles with value of specific charge $\tilde{q} \in (-1.0 \times 10^{13}, 1.0 \times 10^{13})$. Absolute values of such used specific charge values are very low in comparison with $\tilde{q} = 1.111 \times 10^{18}$ corresponding to matter consisting purely of ions of hydrogen.

4 ORBITAL MOTION AND STATIONARY PARTICLES

For corotating orbits with $\Omega_+ = U^{\phi}_+/U^t_+$ the magnetic part of the Lorentz force is attractive for $\tilde{q} > 0$, while for $\tilde{q} < 0$ we observe magnetic repulsion. Inversely oriented electric part of the Lorentz force partially compensates influence of the magnetic component, but for the family of corotating orbits the magnetic component is decisive for the final orientation

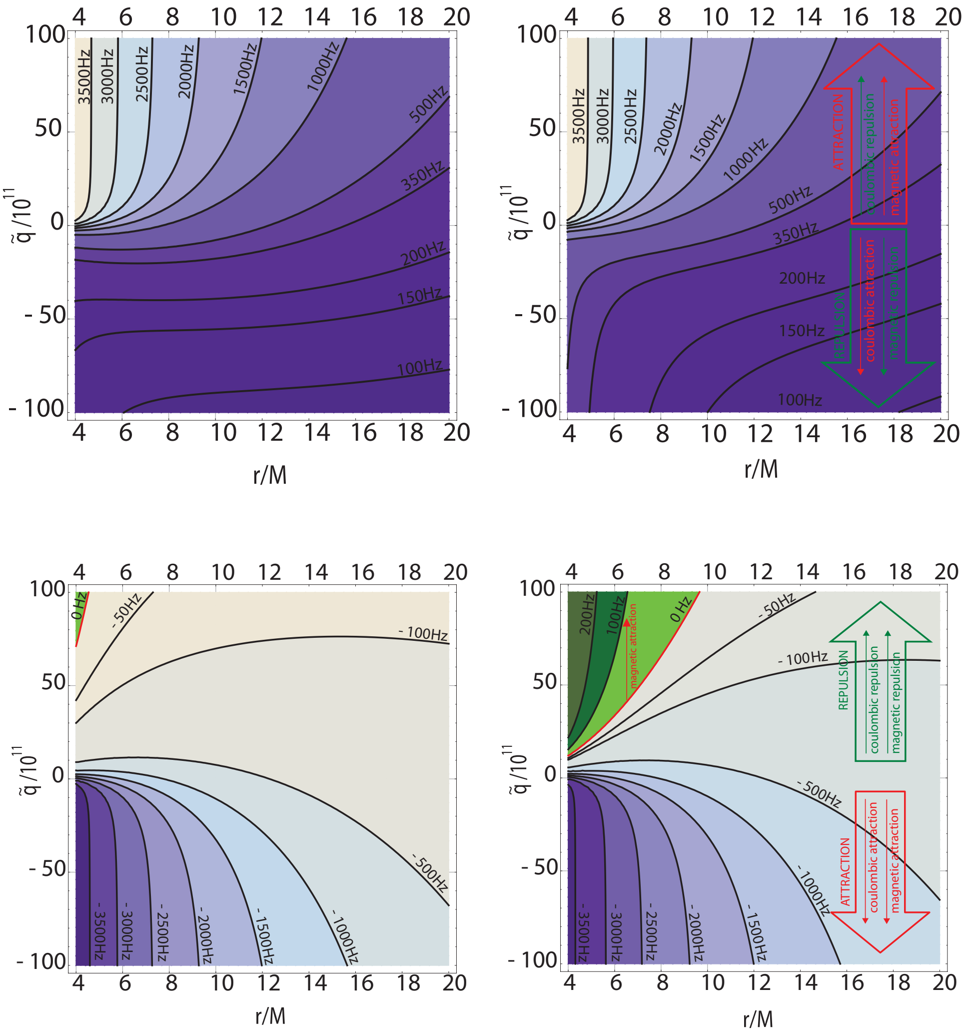

Figure 1. Contour plot of the orbital frequency $\nu = \Omega/2\pi$ as a function of the specific charge $\tilde{q}$ and the radial coordinate constructed for the test neutron star with $M = 1.5\,M_\odot$ and $\mu = 1.06\,\mathrm{x}\,10^{-4}\,\mathrm{m}^2$. *Top left*: Corotating solution for spin $a = 0.05$. *Top right*: Corotating solution for spin $a = 0.3$. *Bottom left*: Counterrotating solution for spin $a = 0.05$. *Bottom right*: Counterrotating solution for spin $a = 0.3$. The electrostatic radii at which the stationary particles are located are given by the red contour line ($\nu = 0\,\mathrm{Hz}$).

of the Lorentz force. Ω_+ increases monotonically with increasing specific charge $\tilde{q}$; in the region of $\tilde{q} < 0$ the repulsive electromagnetic interaction causes decreasing of Ω_+ in comparison with the geodesic orbital frequency Ω_{0+}, while in the attractive region of $\tilde{q} > 0$ there is $\Omega_+ > \Omega_{0+}$. In top panels of Fig. 1 we illustrate behaviour of the orbital angular velocity Ω (related frequency $\nu = \Omega/2\pi$) of orbits described by the corotating solution in dependency on the specific charge $\tilde{q}$ using the test model neutron star with small and extremal values of the spin, $a = 0.05$ and $a = 0.3$.

4.1 Orbital angular velocity of counterrotating solution and stationary particles at electrostatic radius

In the case of the retrograde solution $\Omega_- = U^\phi_-/U^t_+$ the resulting orientation of the Lorentz force is inverse in comparison to the corotating solution. Therefore, $|\Omega_-|$ decreases monotonically with increasing value of the specific charge $\tilde{q}$. For $\tilde{q} < 0$ the attractive Lorentz interaction increases $|\Omega_-|$ in comparison with the geodesic orbital frequency $|\Omega_{0-}|$, while in the repulsive region of $\tilde{q} > 0$ there is $|\Omega_-| < |\Omega_{0-}|$. Nevertheless the relations of both components of the Lorentz force are qualitatively different as compared to the case of the corotating solution.

In the case of the retrograde orbits, both parts of the Lorentz force are oriented identically and the magnetic repulsion and attraction are supported by the electric part of the interaction. Starting from a critical specific charge $\tilde{q}_{\mathrm{es}}$, character of the electromagnetic interaction at the repulsive region enables existence of electrostatic radius $r_{\mathrm{es}}(\tilde{q})$, where the limiting case of the circular orbit with $\Omega_- = 0$ appears. Such particles are static relative to static observers at infinity, with gravitational attraction of the neutron star being compensated by the electric repulsion due to the particle charge. (However, the static particles at the electrostatic radii are rotating relative to the Lense–Thirring spacetime. An analogical situation has been discovered for motion of charged particles in the equatorial plane of the Kerr–Newman geometry (Bálek et al., 1989). In both cases the effect is caused by the combined influence of the frame dragging and the electromagnetic interaction.)

The electric repulsion increases strongly with decreasing radial coordinate. The existence of retrograde solutions for particles with large specific charges orbiting at low radii ($r < r_{\mathrm{es}}(\tilde{q})$, $\tilde{q} \geq \tilde{q}_{\mathrm{es}}$) requires presence of compensating magnetic attraction implying change of the orbital velocity orientation. Then even the retrograde solution determines a special family of corotating (relative to observers at infinity) orbits with relatively low orbital frequency $\Omega_- > 0$ existing paralelly at the same radial coordinate as the orbits of corotating solution demonstrating high $\Omega_+ > 0$.

In bottom panels of Fig. 1 we illustrate behaviour of the orbital angular velocity Ω (related frequency $\nu = \Omega/2\pi$) of orbits described by counterrotating solution in dependency on the specific charge $\tilde{q}$ using test model neutron star with with small and extremal values of the spin, $a = 0.05$ and $a = 0.3$.

5 CONCLUSIONS

The aim of the present paper is to study the influence of the Lorentz force generated by a dipole magnetic field of a slowly rotating neutron star on the equatorial circular motion. We focus on the combined effects of the frame dragging and electromagnetic interaction, representing the frame dragging in the linear approximation of the Lense–Thirring metric. In general, the Lorentz force may be of attractive or repulsive character depending on the sign of orbiting particle's specific charge, the magnetic dipole moment and orbital velocity orientations and the sense of rotation of the neutron star. Surprisingly enough, the combined effect of frame dragging and electro-magnetic interaction implies even in the case of the slow rotation, and in intermediate radii, i.e. radii not close to the gravitational radius, the existence of charged particles being in states appearing static relative to distant static observers. Such particles are located at the so called electrostatic radii. The phenomenon of stationary particles in strong gravity was discovered for the first time in the case of charged particles orbiting the Kerr–Newman black hole, but for ultrarelativistic particles located at close vicinity of the black hole horizon (Bálek et al., 1989). Here we have demonstrated its existence in slightly less exotic conditions around slowly rotating magnetized neutron stars. We shall discuss stability of this kind of motion in a future paper.

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