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Stress-energy tensor of magnetized plasmas in spatially non-symmetric kinetic equilibria

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ABSTRACT
Collisionless astrophysical plasmas at kinetic equilibrium can exhibit geometrical structures characterized by the absence of well-defined global spatial symmetries. Plasmas of this type can arise in the surrounding of compact objects and are likely to give rise to relativistic regimes, being subject to intense gravitational and electromagnetic fields. This paper deals with the investigation of the physical mechanisms related to the occurrence of a non-vanishing equilibrium fluid stress-energy tensor associated with each collisionless species of plasma charged particles belonging to these systems. This permits one to obtain information about the thermal properties of the plasma and to display the related contributions generated by phase-space anisotropies. The issue is addressed from a theoretical perspective in the framework of a covariant Vlasov statistical description, based on the adoption of a relativistic gyrokinetic theory for the single-particle dynamics.

Keywords: collisionless magnetized plasmas – gyrokinetic theory – kinetic equilibria – Vlasov equation – stress-energy tensor

1 INTRODUCTION

The description of the complex phenomenology of plasmas arising in the surrounding of compact objects represents a challenging problem in theoretical astrophysics. In these systems, both single-particle and macroscopic fluid velocities of the plasma can become relativistic, at least in particular subsets of the configuration domains, while space-time curvature effects associated with strong gravitational fields can be relevant. When these circumstances occur, relativistic covariant approaches need to be adopted.

In the following we consider strongly-magnetized collisionless plasmas that can be treated in the framework of a covariant Vlasov–Maxwell formulation and in which single-particle dynamics is relativistic. This allows for both phase-space single-particle as well as electromagnetic (EM) and gravitational collective system dynamics to be consistently taken into account. Within such a description, the fundamental quantity is represented by the species

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kinetic distribution function (KDF) f_s, where s is the species index, whose dynamical evolution is determined by the Vlasov equation.

Astrophysical magnetized plasmas can generate kinetic plasma regimes which persist for long times (with respect to the observer and/or plasma characteristic times), despite the presence of macroscopic time-varying phenomena of various origin, such as flows, non-uniform gravitational/EM fields and EM radiation, possibly including that arising from single-particle radiation-reaction processes (Hazeltine and Mahajan, 2004; Cremaschini and Tessarotto, 2011). For collisionless plasmas, these states might actually correspond to some kind of kinetic equilibrium which characterizes the species KDFs. This is realized when the latter distributions are all assumed to be functions only of the single-particle adiabatic invariants. Therefore, in this sense kinetic equilibria may arise also in physical scenarios in which macroscopic fluid fields (e.g. fluid stress-energy tensor) and/or the EM field might be time dependent when observed from an observer reference frame.

For non-relativistic axisymmetric systems, the subject was treated in Cremaschini et al. (2010, 2011); Cremaschini and Stuchlík (2013); Cremaschini et al. (2013b), where kinetic equilibria were investigated for collisionless magnetized plasmas subject to stationary or quasi-stationary EM and gravitational fields. A number of peculiar physical properties have been pointed out in this reference, which range from quasi-neutrality, the self-generation of equilibrium EM fields and the production of macroscopic azimuthal and poloidal flow velocities, together with the occurrence of temperature and pressure anisotropies. Further interesting developments concern, however, a more general physical setting in which some of the relevant symmetry properties characteristic of the equilibria indicated above, may be in part lost. These include both spatially non-symmetric kinetic equilibria in which energy is conserved (Cremaschini and Tessarotto, 2013) as well as energy-independent kinetic equilibria (Cremaschini et al., 2013a) in which a continuous spatial symmetry of some kind still survives.

Extension of these results to relativistic plasmas of the type indicated above has been established in recent contributions (Cremaschini et al., 2014b,a). In particular, in Cremaschini et al. (2014b) kinetic equilibria of relativistic collisionless plasmas in the presence of non-stationary EM fields have been addressed, while Cremaschini et al. (2014a) dealt with the covariant formulation of spatially non-symmetric kinetic equilibria in magnetized plasmas and the determination of the physical mechanisms responsible for the occurrence of a non-vanishing 4-flow. This concerns systems characterized by non-axisymmetric morphologies as far as the behaviour of both the EM and fluid fields is concerned, while the background gravitational field can still be allowed to exhibit space-time symmetries of some kind (e.g. to be defined with respect to the distant observer coordinate system).

In both these cases, the theory has required the development of a systematic non-perturbative formulation of covariant gyrokinetic theory (Beklemishev and Tessarotto, 1999, 2004) for the appropriate Lagrangian variational description of single-particle dynamics in relativistic plasma regimes. The GK theory in fact provides the appropriate framework for the determination of exact and adiabatic phase-space particle conservation laws. In particular, the novel GK theory presented in Cremaschini et al. (2014b,a) permits one to identify a non-perturbative representation of the particle magnetic moment, which is shown to be conserved even when global space-time symmetries may be absent. In addition, in Cremaschini et al. (2014a) a perturbative representation of the exact GK theory has been developed based on the so-called Larmor-radius expansion, allowing the magnetic moment to be evaluated asymptotically as an adiabatic invariant with prescribed accuracy and the higher-order Larmor-radius corrections to its expression to be consistently determined.

The adiabatic conservation of the single-particle magnetic moment is a distinctive feature of collisionless magnetized plasmas. Indeed, for both relativistic and non-relativistic systems, the magnetic moment is the primary source of temperature anisotropy, while for spatially non-symmetric configurations it is essential in order to generate macroscopic plasma flows along both the parallel and perpendicular directions with respect to the local magnetic field.

Based on these premises and extending the research pursued in Cremaschini et al. (2014a), the purpose of the present work is to investigate the physical mechanisms which determine the properties of the equilibrium fluid stress-energy tensor $T^{\mu\nu}$ associated with relativistic collisionless plasmas in spatially non-symmetric configurations. This provides the correct equilibrium fluid closure condition for these systems, which carries information about the thermal properties of the plasma and the different contributions generated by phase-space anisotropies. The issue is addressed from a theoretical perspective in the framework of a covariant Vlasov statistical description of magnetized plasmas, based on the adoption of the covariant GK theory for the single-particle dynamics earlier developed. In particular, the main goals of the study are as follows:

(1) To summarize the main features of the GK theory and provide the perturbative representation of the relativistic magnetic moment.

(2) To outline the method for the construction of kinetic equilibria, providing an explicit representation of the species KDF in the form of a generalized Gaussian distribution.

(3) To calculate the expression of the stress-energy tensor and to show that this is generally non-isotropic. It is pointed out that this feature arises primarily from the conservation of the magnetic moment carried by the equilibrium KDF. The asymptotic expression of the magnetic moment correct up to first order in the Larmor-radius expansion is adopted for this task, which permits an analytical estimate of the corresponding distinctive contributions to the stress-energy tensor.

2 NON-PERTURBATIVE GK THEORY

In this section we summarize the main results concerning the non-perturbative formulation of the covariant GK theory, treating particles as point-like having specific charge $q \equiv Ze/M_0c^2$, with M_0 being the mass of the species component particles, and moving in a prescribed background metric tensor $g_{\mu\nu}(r)$ and EM 4-potential A_μ. The GK theory is obtained by introducing an extended phase-state transformation of the form

$$\mathbf{x} \equiv \left(r^\mu, u^\mu\right) \leftrightarrow \mathbf{z}' \equiv \left(\mathbf{y}', \phi'\right) , \tag{1}$$

where ϕ' is the gyrophase angle, $\mathbf{z}'$ is the GK state and $\mathbf{y}'$ is a suitable 7-component vector. The GK state $\mathbf{z}'$ is constructed in such a way that its equations of motion are gyrophase independent, namely $\mathrm{d}\mathbf{z}'/\mathrm{d}s \equiv \mathbf{F}(\mathbf{y}', s)$, where $\mathbf{F}$ is a suitable vector field. A non-perturbative covariant GK theory is established by introducing the extended local transformation of the type

$$r^{\mu} = r'^{\mu} + \rho_1'^{\mu}, \tag{2}$$
$$u^{\mu} = u'^{\mu} \oplus \nu_1'^{\mu}, \tag{3}$$

denoted as extended guiding-center transformation, where $\rho_1'^{\mu} = \rho_1'^{\mu}\left(r'^{\mu}, u'^{\mu}\right)$ and $\nu_1'^{\mu} = \nu_1'^{\mu}\left(r'^{\mu}, u'^{\mu}\right)$ are suitably prescribed in terms of $\left(r'^{\mu}, u'^{\mu}\right)$. Here r'^{μ} is the guiding-center position 4-vector, with primed quantities denoting dynamical variables which are evaluated at r'^{μ}. Thus, $\rho_1'^{\mu}$ is referred to as the relativistic Larmor 4-vector, while both u^{μ} and u'^{μ} are by construction 4-velocities, so that $u^{\mu}u_{\mu} = u'^{\mu}u'_{\mu} = 1$, with $\oplus$ denoting the relativistic 4-velocity composition law. Notice that by construction $u_1'^{\mu} \equiv u'^{\mu} \oplus \nu_1'^{\mu}$ is necessarily a 4-velocity, although $\nu_1'^{\mu}$ is not necessarily so.

The guiding-center transformation (2) and (3) are required to fulfil the equation

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(r'^{\mu} + \rho_1'^{\mu}\right) = u'^{\mu} \oplus \nu_1'^{\mu}, \tag{4}$$

which relates the transformed physical velocity to the rate of change of the displacement vector $r'^{\mu} + \rho_1'^{\mu}$.

The 4-velocity u'^{μ} is projected along the EM-tetrad of unit 4-vectors $\left(a'^{\mu}, b'^{\mu}, c'^{\mu}, d'^{\mu}\right)$ evaluated at the guiding-center position (guiding-center EM-tetrad, see Cremaschini et al. (2014a)), yielding the representation

$$u'^{\mu} \equiv u_0'a'^{\mu} + u_{\|}'b'^{\mu} + w'\left[c'^{\mu}\cos\phi' + d'^{\mu}\sin\phi'\right], \tag{5}$$

where $u_0' = \sqrt{1 + u_{\|}'^2 + w'^2}$. The derivation of the GK equations of motion and of the related conservation laws follow standard procedures in the framework of variational Lagrangian approach. In particular, provided the two transformations (1), (2) and (3) actually exist, i.e. are invertible, one obtains the following expression for the non-perturbative representation of the particle magnetic moment m':

$$m' = \left\langle \frac{\partial \rho_1'^{\mu}}{\partial \phi'}\left[\left(u'_{\mu} \oplus \nu'_{1\mu}\right) + qA_{\mu}\right]\right\rangle_{\phi'}, \tag{6}$$

which is by construction a 4-scalar.

3 PERTURBATIVE GK THEORY

The perturbative GK theory is obtained by introducing a perturbative method based on the introduction of the dimensionless Larmor-radius parameter, namely the frame-invariant ratio $\varepsilon \equiv r_L/L \ll 1$, to be considered as an infinitesimal. Here r_L is the Larmor-radius 4-vector, while L is a suitable characteristic invariant length of the system. Then we introduce the assumption that both $\rho_1'^{\mu}$ and $\nu_1'^{\mu}$ are considered as infinitesimals and are represented in terms of the power series

$$\varepsilon\rho_1'^{\mu} = \varepsilon r_1'^{\mu} + \varepsilon^2 r_2'^{\mu} + \cdots, \tag{7}$$

$$\varepsilon v_1'^{\mu} = \varepsilon v_1'^{\mu} + \varepsilon^2 v_2'^{\mu} + \cdots \tag{8}$$

Similarly, the 4-vector potential is Taylor-expanded in ε around the guiding-center position r'^{μ}. Then, introducing these expressions in the GK Lagrangian differential form and evaluating its gyrophase average yields the following perturbative representation for the particle magnetic moment m':

$$m' = \mu' + \varepsilon \mu_1' + O\left(\varepsilon^2\right). \tag{9}$$

In detail, here μ' is the leading-order contribution given by

$$\mu' = \frac{w'^2}{2qH'}, \tag{10}$$

where H' is the magnetic field strength in the EM-tetrad reference frame. Furthermore, μ_1' is the first-order contribution, which can be written in compact form as

$$\mu_1' = \mu' \left(u_0' \Delta'_{u_0'}\left(r'\right) + u_\parallel' \Delta'_{u_\parallel'}\left(r'\right) \right) + \mu' w' \Delta'_{w'}\left(r'\right), \tag{11}$$

where the 4-scalar coefficients $\Delta'_{u_0'}(r')$, $\Delta'_{u_\parallel'}(r')$ and $\Delta'_{w'}(r')$ are only position-dependent. We omit to calculate here their precise expression as this is not needed for the subsequent developments.

Some important features must be pointed out regarding the asymptotic representation of the magnetic moment given above:

(1) The contribution μ_1' is linearly proportional to the leading-order magnetic moment μ'.

(2) Provided $\Delta'_{u_0'}\left(r'\right)$ and $\Delta'_{u_\parallel'}\left(r'\right)$ are non-zero, the first-order magnetic moment μ_1' contains linear velocity dependences in terms of u_0' and $u_\parallel'$.

(3) The contribution proportional to u_0' is an intrinsically-relativistic effect since u_0' is related to the other components of the 4-velocity by means of a square-root dependence. Concerning the dependences in terms of $u_\parallel'$, we notice that besides the linear one, there is an additional intrinsically relativistic one appearing through u_0'.

4 RELATIVISTIC KINETIC EQUILIBRIA

In this section the construction of relativistic spatially non-symmetric kinetic equilibria for collisionless plasmas in curved space-time is considered. To reach the target, the method of invariants is implemented, which consists in expressing the species KDF in terms of exact or adiabatic single-particle invariants. In the present case the latter is identified with the set (P_0, m'), where P_0 is the conserved momentum conjugate to the ignorable time coordinate, as it follows from the stationarity condition. Therefore one can always represent the species equilibrium KDF in the form $f_s = f_{*s}$, with

$$f_{*s} = f_{*s}\left(\left(P_0, m'\right), \Lambda_*\right) \tag{12}$$

being a smooth strictly-positive function of the particle invariants only which is sum-able in velocity-space. Concerning the notation, in Eq. (12) $\left(P_0, m'\right)$ denote explicit functional dependences, while Λ_* denotes the so-called structure functions (Cremaschini et al., 2011), namely functions suitably related to the observable velocity moments of the KDF. In the following for simplicity the particular choice $\Lambda_* = \text{const}$ is adopted. Notice that in the following, for simplicity of notation but without possible misunderstandings, we omit to indicate the index s in the set of structure functions entering each species KDF.

For the sake of illustration, we consider here a specific realization of each species KDF f_{*s} in terms of a generalized Gaussian distribution. To this aim, we denote with $P_\mu = (u_\mu + qA_\mu)$ the particle generalized 4-momentum in the observer (laboratory) frame which is characterized by the co-moving 4-velocity U^μ, so that in this frame one has simply $U^\mu = (1, 0, 0, 0)$. As a consequence, in the observer frame $P_\mu U^\mu = P_0$ (rest energy), which is a conserved 4-scalar by assumption. Therefore, it follows that f_{*s} can be identified with the 4-scalar

$$f_{M*s} = \beta_* e^{-P_\mu U^\mu \gamma_* - m'\alpha_*}\,, \tag{13}$$

where the structure functions are represented by the set of 4-scalars fields $\{\Lambda_*\} \equiv \{\beta_*, \gamma_*, \alpha_*\}$. From the physical point of view, here β_* is related to the plasma 4-flow, or equivalently the plasma number density when measured in the fluid co-moving frame, while γ_* and α_* are related to the temperature anisotropy. We stress that the representation of the KDF in Eq. (13) is still exact, in the sense that no asymptotic approximations have been introduced there, so that the magnetic moment m' in the exponential factor must be given by its non-perturbative representation by Eq. (6).

In order to determine explicitly the 4-velocity moments of f_{M*s}, the magnetic moment m' must be preliminarily evaluated at the actual particle position by means of an inverse guiding-center transformation. When the latter is applied to the perturbative representation of m' given by Eq. (9), this leads, with the same accuracy, the following expression for the magnetic moment m:

$$m = \mu + \varepsilon\mu_1 + \varepsilon\delta_{(\mu)}\,. \tag{14}$$

Here $\mu \equiv w^2/2qH$ is the leading-order contribution, μ_1 is the first-order term which coincides with that in Eq. (11) when evaluated at the particle position, while the $O(\varepsilon)$ correction $\delta_{(\mu)} = \delta_{(\mu)}\left(r, u_\parallel, \mu, \phi\right)$ contains explicit gyrophase dependences and originates from the inverse guiding-center transformation applied to μ'. Finally, in terms of Eq. (14) and neglecting second-order corrections in ε, the equilibrium species KDF (13) becomes

$$f_{M*s} = \beta_* e^{-P_{\mu R} U_R^\mu(r)\gamma_* - \mu\alpha_*}\left[1 - \left(\varepsilon\mu_1 + \varepsilon\delta_{(\mu)}\right)\alpha_*\right], \tag{15}$$

where all quantities are represented in the EM-tetrad with origin at the actual particle position. Thus, $P_{\mu R}$ is the canonical momentum and $U_R^\mu(r)$ the 4-velocity corresponding to U^μ, both expressed in the same EM-reference frame.

5 THE STRESS-ENERGY TENSOR

As shown in Cremaschini et al. (2014a), the phase-space functional dependences contained in the KDF f_{M*s} given by Eq. (15) give rise to corresponding fluid equilibria characterized by non-uniform 4-flows $N^{\mu}(r)$. In this section we consider another velocity-moment of the KDF. In particular we investigate the form of the fluid stress-energy tensor $T^{\mu\nu}(r)$, in order to prove that this is generally non-isotropic and to identify the different phase-space contributions that determine its form.

In detail, the plasma stress-energy tensor $T^{\mu\nu}(r)$ is defined as

$$T^{\mu\nu}(r) = \sum_{s} T_s^{\mu\nu}, \tag{16}$$

where $T_s^{\mu\nu}$ denotes the generic species stress-energy tensor given by the 4-velocity integral

$$T_s^{\mu\nu}(r) = 2M_o c^2 \int \sqrt{-g}\, \mathrm{d}^4 u\, \Theta\left(u^0\right) \delta\left(u^{\mu} u_{\mu} - 1\right) u^{\mu} u^{\nu} f_{M*s}. \tag{17}$$

In the previous expression the Dirac-delta takes into account the kinematic constraint for the 4-velocity when performing the integration, while $\sqrt{-g}$ is the square-root of the determinant of the background metric tensor. Invoking the EM-tetrad representation for the 4-velocity, the integral can be reduced to

$$T_s^{\mu\nu}(r) = M_o c^2 \int \frac{\sqrt{-g}\, \mathrm{d}^3 u}{\sqrt{1 + u_{\parallel}^2 + w^2}} u^{\mu} u^{\nu} f_{M*s}. \tag{18}$$

When the previous integral is evaluated with respect to the EM-tetrad reference frame, then locally $\sqrt{-g} = 1$, thanks to the principle of equivalence. In such a framework, one can introduce the cylindrical coordinates in the velocity space:

$$\int \mathrm{d}^3 u \to \int_0^{2\pi} \mathrm{d}\phi \int_0^{+\infty} w\, \mathrm{d}w \int_{-\infty}^{+\infty} \mathrm{d}u_{\parallel}, \tag{19}$$

where $u_{\parallel}$ and w coincide with the scalar components of the 4-velocity analogous to those entering Eq. (5) when expressed at the actual particle position and in terms of which the KDF is represented. Hence, in the EM-tetrad frame the integral becomes finally

$$T_s^{\mu\nu}(r) = M_o c^2 \int_0^{2\pi} \mathrm{d}\phi \int_0^{+\infty} w\, \mathrm{d}w \int_{-\infty}^{+\infty} \mathrm{d}u_{\parallel} \frac{u^{\mu} u^{\nu} f_{M*s}}{\sqrt{1 + u_{\parallel}^2 + w^2}}. \tag{20}$$

Although its explicit evaluation can be in principle carried out numerically, in this study we are interested in evaluating its qualitative features in terms of an analytical analysis.

First we notice that, once u^{μ} is represented in the EM-tetrad in terms of the basis formed by $(a^{\mu}, b^{\mu}, c^{\mu}, d^{\mu})$, the same 4-vectors also identify the tensorial components of $T_s^{\mu\nu}(r)$, which are generally position-dependent. Once the expression of $T_s^{\mu\nu}(r)$ is known in such a frame in terms of the EM-tetrad, its representation can then be determined in arbitrary reference frames (i.e. coordinate-systems). A second feature to mention is that, by construction, the tensor $T_s^{\mu\nu}(r)$ is symmetric, with non-vanishing diagonal components. Additional properties can be inferred when the representation (15) is adopted. In particular:

(1) The leading-order contribution to $T_s^{\mu\nu}(r)$ is generated by the velocity dependences contained in the exponential factor, carried respectively by the 4-scalars $P_{\mu R}U_R^{\mu}(r)$ and μ. This determines the leading-order fluid closure condition of the system and the tensorial equation of state of the plasma, to be generally of non-polytropic type for these systems, providing information about the thermal state of the kinetic equilibrium. Since the magnetic moment μ depends only on the component w of the 4-velocity, the separate contributions of the leading-order tensor are different. In the EM-tetrad frame the tensor acquires its simplest representation and becomes diagonal at this order, so that an anisotropy clearly arises in analogy with the non-relativistic solution. This realizes the so-called temperature anisotropy.

(2) The first-order term generated by μ_1 contains three separate contributions, according to the expression (11). In particular, the terms proportional to μu_0 and μw yield corrections to the leading-order solution, thus affecting only the diagonal terms and exhibiting the same kind of anisotropy when the tensor is expressed in the EM-tetrad frame. Instead, more interesting, the term proportional to $\mu u_{\parallel}$ is odd in the parallel component $u_{\parallel}$, and therefore it generates non-vanishing contributions in the tensorial directions (hyperplane) $a^{\mu}b^{\nu}$, so that in the EM-tetrad frame this provides non-diagonal contributions. It is important to stress that the latter feature is a unique consequence of the first-order correction to the magnetic moment, which is missing in the leading-order solution, implying that, for consistency, the first-order perturbations cannot generally be neglected.

(3) Similar considerations apply also to the first-order term associated with the correction $\delta_{(\mu)}$ to the magnetic moment. In view of the general form of its functional dependence and its explicit gyrophase dependence, this term is expected to possibly contribute to all components of the stress-energy tensor, thus extending the number of possible non-vanishing off-diagonal terms (in the EM-tetrad frame).

Finally, a comment is in order concerning the spatial dependences in terms of r^{μ} arising in $T_s^{\mu\nu}(r)$. In the present case in which the structure functions are constant, non-trivial configuration-space dependences still arise due to the following physical effects: 1) the explicit dependence in terms of the 4-scalar $A_{\mu}U_R^{\mu}(r)$ associated with P_{μ}; 2) the functional form of the 4-vector $U_R^{\mu}(r)$, which is determined by the boost transformation; and finally 3) the spatial dependences appearing in the 4-scalars μ, μ_1 and $\delta_{(\mu)}$ occurring due to the inhomogeneities of the background EM field.

6 CONCLUSIONS

In this study the physical properties of the stress-energy tensor associated with relativistic magnetized collisionless plasmas belonging to spatially non-symmetric configurations have been investigated. An analytical approach has been adopted to address the problem. The theory has been developed in the framework of a covariant Vlasov statistical description, based on the adoption of a relativistic gyrokinetic theory for the single-particle dynamics.

A fundamental element is the calculation of the relativistic single-particle magnetic moment, which represents an adiabatic invariant of prescribed accuracy. A perturbative solution correct through first-order in the Larmor-radius expansion has been determined in this context. The expression of the magnetic moment is fundamental for the consistent realization of kinetic equilibria, obtained here in terms of generalized Gaussian-like distributions. In addition, it has been shown that the same adiabatic invariant represents the main source of phase-space anisotropies which ultimately give rise to a non-isotropic stress-energy tensor. When the latter is evaluated in the EM-tetrad frame, the occurrence of a leading-order temperature anisotropy is manifest, while non-vanishing off-diagonal first-order corrections are characteristic of these systems.

The results obtained here are useful in order to display the thermal properties of spatially non-symmetric plasmas and provide the appropriate theoretical framework for a better understanding of the statistical features of astrophysical collisionless plasmas arising in relativistic regimes and subject to the simultaneous action of intense gravitational and electromagnetic fields.

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