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Revealing general relativity effects from accretion events near a supermassive black hole

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ABSTRACT
Accretion onto black holes often proceeds via an accretion disc or a temporary disc-like pattern. Variability features, observed in the light curves of such objects, and theoretical models of accretion flows suggest that inner accretion discs are inhomogeneous and clumpy. We discuss the general relativity effects acting on the radiation signal from the inner accretion flow. To this end we consider the radiation flux and polarization properties originating from a blob of gas near a rotating black hole. The predicted observed polarization at infinity is changed from its local value due to strong gravity and fast orbital motion. Different processes can produce the observed pattern: in the context of Sgr A* flares, the synchrotron mechanism and the inverse Compton upscattering appear to be the most likely mechanisms. The energy dependence of the changing degree and angle of polarization should allow us to discriminate between rotating (Kerr) and a non-rotating (Schwarzschild) black hole.

Keywords: Black holes – Galactic Center – Relativity

1 INTRODUCTION

Polarization of light originating from different regions of a black hole accretion disc and detected by a distant observer is influenced by strong gravitational field near a central black hole. A 'spotted' accretion disc is a useful model of an interface of such an inhomogeneous medium, assuming that there is a well defined boundary between the disc interior and the outer, relatively empty space. Relativistic corrections to a signal from orbiting spots can lead to large rotation in the plane of observed X-ray polarization. When integrated over an extended surface of the source, this can diminish the observed degree of polarization. Such effects are potentially observable and can be used to distinguish among different models of the source geometry and the radiation mechanisms responsible for the origin of the polarized signal. The polarization features show specific energy and time dependencies which can indicate whether a black hole is present in a compact X-ray source.

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Practical implementation of the idea, originally proposed in the late 1970s (Connors and Stark, 1977; Pineault, 1977), is a challenging task because the polarimetric investigations need a high signal-to-noise ratio. Also, the interpretation of the model results is often very sensitive to the assumptions about the radiation transfer in the source and the geometrical shape and orientation of the emission region. Nevertheless, the technology has achieved significant advances since the 1980s and reached a mature state.

We assume that the gravitational field is described by a rotating black hole, and so the Kerr metric is the right model for the gravitational field. Using a constant of motion along null geodesics in the Kerr metric (Walker and Penrose, 1970), one can determine the change of polarization angle along light rays. While the polarization degree is scalar and the gravitational influence of the black hole does not change it along the null geodesic (which is identical with the light ray path in the geometrical optics approximation), the observed polarization angle is affected. The calculations show that general-relativistic effects can cause large rotation of the polarization angle and produce significant fluctuations in the degree of observed polarization due to gravitational bending.

Although the geometrical effects of strong gravitational fields act on photons independently of their energy, the intrinsic emissivity of accretion discs and the influence of turbulent magnetic fields, intervening via Faraday rotation, are indeed energy dependent. As a result, the variability amplitudes of both the polarization degree the polarization angle must be energy dependent quantities as well. Furthermore, the signal resolved with respect to both energy and polarization (i.e. the spectro-polarimetric information) can probe different regions of the accretion disc due to radially varying temperature.

These dependencies suffer from some degeneracy, which can be avoided with time-resolved observations. Namely, if the source is an orbiting spot near a black hole, the time variation of the observed signal reflects the presence of strong gravity effects (Connors et al., 1980; Bao et al., 1997; Murphy et al., 2009). A related problem of spots rotating on the surface of a compact star was also investigated (Viironen and Poutanen, 2004).

On the whole, there are some similarities as well as differences between the expected manifestation of GR polarization changes in X-rays and in other spectral bands, such as the infrared region. We will mention these interrelations and point out that the near-infrared polarization measurements of the radiation flares from the immediate vicinity of the horizon are already now available for Sagittarius A* supermassive black hole in the Galaxy center (Meyer et al., 2006; Zamaninasab et al., 2008).

2 POLARIZATION FROM BLACK HOLE ACCRETION DISCS

It was realized some four decades ago that X-ray polarization studies could provide decisive clues to the physics of accreting compact objects (Angel 1969; Bonometto et al. 1970; Lightman and Shapiro 1975; Rees 1975). In the non-relativistic regime, a conceptually similar problem was discussed by Rudy (1978) and Fox (1994), who studied the polarization of star light caused by an ionized circumstellar shell of free electrons.

Wave fronts of light propagating near a rotating black hole do exhibit the frame dragging effect. On the other hand, the wave fronts do not depend on polarization (in geometrical optics approximation). Therefore, the impact of strong gravity on observed polarization comes in a somewhat complicated manner, through the interplay of light-bending, aberration and the Doppler effect.

In the context of accretion discs the effect of electron scattering atmosphere has been also often invoked. Further, polarization of the Comptonized radiation of accretion discs was examined as a function of various model parameters, such as the optical thickness of the disc medium, energy of scattered photons and directional angle of the emission (Stark 1981; Williams 1984; Sunyaev and Titarchuk 1985).

Two basic schemes were proposed as being relevant for the X-ray polarization from the inner accretion disc. Firstly, the accretion disc surface lies below the scattering atmosphere and acts as a source of seed photons. Polarization of thermal radiation from a black hole accretion disc was also studied (Laor et al., 1990). The reason for polarization is that photons from the disc are scattered by electrons within the disc atmosphere. Linear polarization should arise in the disc local co-rotating reference frame. This situation is expected to be particularly relevant for Galactic black hole candidates whose discs exhibit phases of strong multi-blackbody thermal radiation dominating over other spectral components. The early investigations were recently put forward by several groups (Dovčiak et al., 2008; Li et al., 2009). Secondly, Matt (1993) and Dovčiak et al. (2004) examined the polarimetric consequences of a specific model of a lamp-post illuminated accretion disc. Within this scheme the number of reflected (polarized) photons is proportional to the incident flux arriving from the primary source.

Relativistic effects would be even more prominent and unique if one could include the higher-order, gravitationally bent light rays (Horák and Karas 2006) and the effects of disc self-irradiation (Schnittman and Krolik, 2009). In fact, the latter authors argue that the self-irradiation effects can be surprisingly important for polarization measurements.

3 TIME-VARYING POLARIZATION FROM AN ORBITING SPOT

The model of an orbiting bright spot (e.g. Cunningham and Bardeen, 1972; Broderick and Loeb, 2006; Meyer et al., 2006; Noble et al., 2007) has been fairly successful in explaining the observed modulation of various accreting black hole sources. Certainly not all variability patterns can be explained in this way, however, the scheme is general enough to be able to capture also the effects of spiral waves and similar kind of transient phenomena that are expected to occur in the disc (Tagger et al. 1990, 2006; Karas et al. 2001). It can be argued that the spot lightcurves can be phenomenologically understood as a region of enhanced emission that performs a co-rotational motion near above the innermost stable circular orbit (ISCO). For example, within the framework of the flare-spot model (Czerny et al., 2004) the spots are just regions of enhanced emission on the disc surface rather than massive clumps that could suffer from fast decay due to shearing motion in the disc. The observed signal is modulated by relativistic effects. According to this idea, Doppler and gravitational lensing influence the observed radiation flux and this can be computed by ray-tracing methods. Such an approach has been extended to compute also strong gravity effects acting on polarization properties (Dovčiak et al., 2004).

To summarize our model, we assumed a Keplerian geometrically thin and optically thick disc around Kerr black hole. A spot is supposed to be intrinsically polarized by different mechanisms – either by reflection of a primary flare on the disc surface or by synchrotron

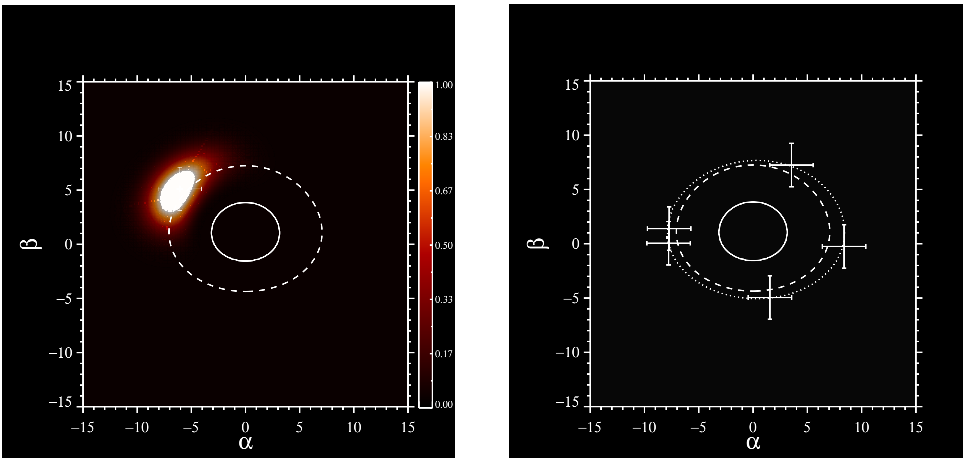

Figure 1. *Left*: A snapshot of a spot orbiting at constant radius $r = 1.1\, r_{\mathrm{ISCO}}$. The image is shown in the observer plane (α, β), for a non-rotating black hole observed at a moderate view angle, $\theta_{\mathrm{O}} = 45$ deg. The horizon radius (*solid curve*) and the ISCO (*dashed curve*) are shown for the reference. *Right*: Trajectory of the image centroid during one revolution of the spot. The wobbling position of the image centroid is indicated by crosses at different moments along the image track (*dotted curve*).

emission originating from an expanding blob, as detailed below, or within the framework of the accretion–ejection scheme (see also Melrose, 1971; Eckart et al., 2008; Huang et al., 2008 and references cited therein). The blob represents a rotating surface feature in the accretion flow. It shares the bulk orbital motion of the underlying medium at sufficiently large radii above the ISCO, gradually decaying due to differential rotation of the disc.

We have applied different prescriptions for the local polarization (see Dovčiak et al., 2006; Meyer et al., 2006; Zamaninasab et al., 2008 for the detailed description of the model set-up in the individual cases that we investigated). For example, one set of models assumes the local emission to be polarized either in the direction normal to the disc plane, or perpendicular to the toroidal magnetic field. Obviously, in the case of partial local polarization the observed polarization signal will be diluted by an unpolarized fraction, and so the polarization degree of the final signal will be proportionally diminished. In another set of models we assumed a lamp-post illuminated spot as the source of spot polarization by reflection. For the spot shape we first assumed the spot does not change its shape during its orbit, but then we also consider the spot decay with time. The relativistic effects can be clearly identified and understood with these simple (and astrophysically unrealistic) toy models, as they produce visible signatures in the observed polarization properties.

General relativistic effects present in our model can be split into two categories. Firstly, it is the symmetry breaking between the approaching and the receding part of the spot orbit. Doppler beaming as well as the light focusing contribute to the change of the observed flux, especially at high view angles when the spot orbit is seen almost edge-on. Notice that the Doppler boosting effect is off phase with respect to the light focusing effect, roughly by 0.25 of the full orbit at the corresponding radius. Here, the precise number depends on the

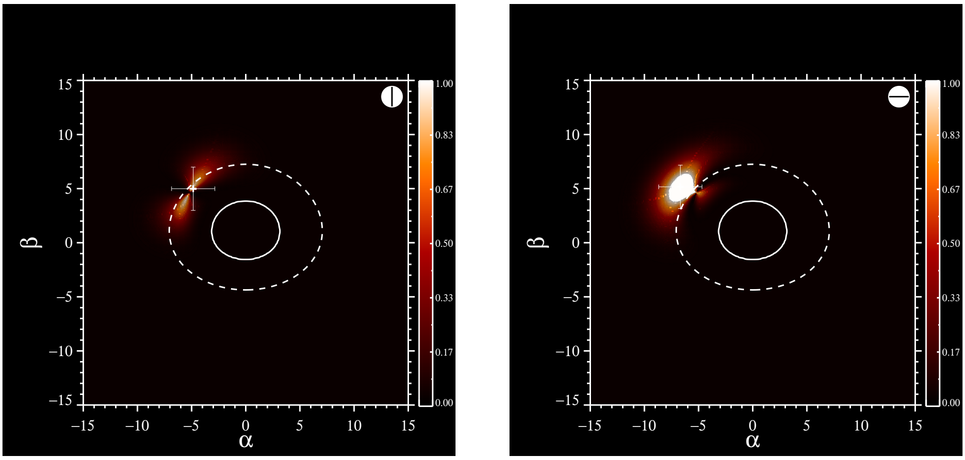

Figure 2. As in the left panel of the previous figure, but now the spot emission is assumed to be intrinsically polarized and recorded in two polarization channels, rotated by 90 degrees with respect to each other (Zamaninasab et al., 2010).

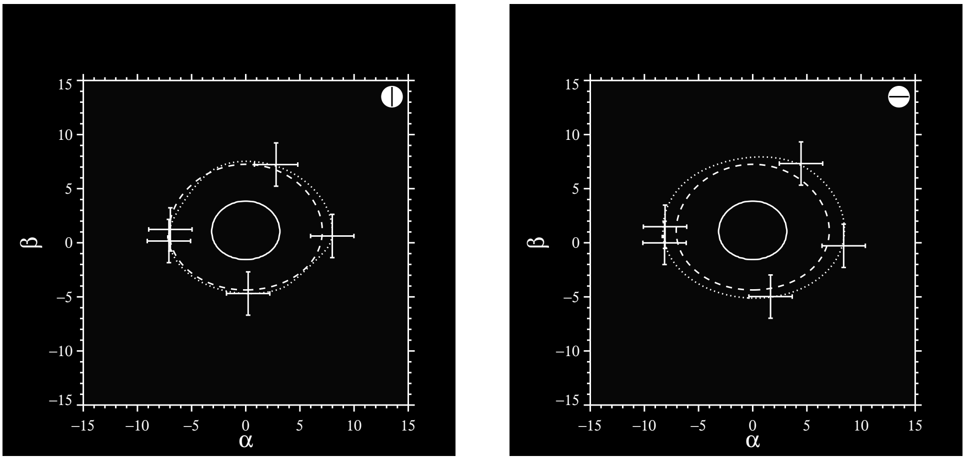

Figure 3. Trajectory of the image centroid during one revolution of the spot corresponding to the previous figure.

black hole spin; it also depends on the inclination through the finite light-travel time from different parts of the spot orbit towards the observer. Also, higher order images could be important in case of almost edge-on view of the spot.

Secondly, rotation of the polarization plane along the photon trajectory also plays a role. This effect is particularly strong for small radii of the spot orbit, in which case a critical point occurs (Dovčiak et al., 2008). The observed polarization angle exhibits just a small wobbling around its principal direction when the spot radius is above the critical point, whereas it starts turning around the full circle once the radius drops below the critical

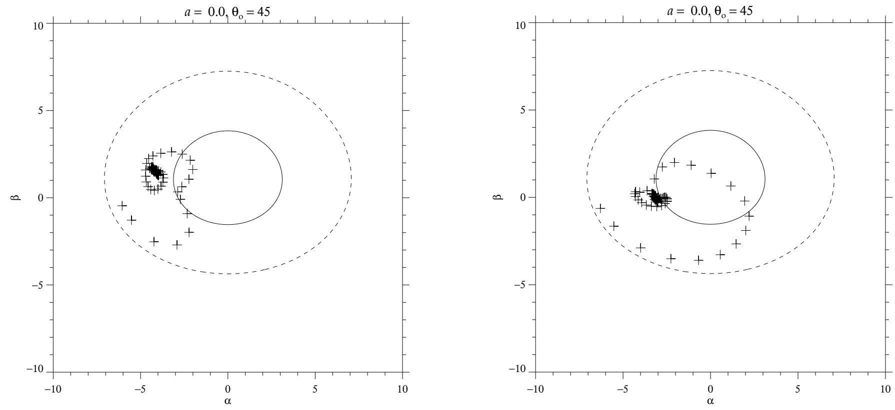

Figure 4. Similar to Fig. 3, but now the spot is supposed to become elongated and eventually decay due to the shearing motion in the accretion disc. Also the disc itself also to part of the emitted radiation in this example. The case of a rotating black hole, $a = 0.5\,M$, seen at $\theta_\mathrm{o} = 45$ deg. The final track of the centroid image was extracted, taking into account both the spot and the underlying disc contributions. As a result of this model set-up, the centroid wobbles slightly off the actual center of the system, and the centroid motion evolves as the spot gradually disappears. Each orbit remains just above the ISCO and the image of the corresponding track settles down within one or two full orbits.

one. Notice that the exact location of the critical point depends on the black hole angular momentum, in principle allowing us to determine its value.

However, a caveat (and a third point on the list) is caused by sensitivity of the critical radius to the special relativistic aberration effects, especially at small view angles (i.e. when the spot is seen almost along the rotation axis). This means that the moment when the observed polarization angle starts rotating is sensitive to the underlying assumption of a perfectly planar geometry of the disc surface.

Obviously the turbulent magnetic fields will play a role in diminishing the observed polarization degree, and that part has been neglected in the present contribution. It is worth noticing, however, that the impact of Faraday rotation on the observed polarization decreases with the square of photon energy, and hence it is less restricting in the X-ray band.

By combining the above-mentioned effects together, Dovčiak et al. (2006) have shown that the observed polarization degree is expected to decrease (in all their models) mainly in that part of the orbit where the spot moves close to the region where the photons are emitted perpendicularly to the disc. In this situation the polarization angle changes rapidly. The decrease in the observed polarization degree for the local polarization perpendicular to the toroidal magnetic field happens also in those parts of the orbit where the magnetic field points approximately along light ray.

However, for the more realistic models the resulting polarization shows a much more complex behaviour. Among persisting features is the peak in polarization degree for the extreme Kerr black hole for large inclinations, caused by the lensing effect at a particular position of the spot in the orbit where the polarization angle is changed. This is not visible in the Schwarzschild case.

The X-ray polarization lightcurves and spectra are still to be taken by future missions, but one may envision even a more challenging goal connected with imaging of the inner regions of accreting black hole sources. Obviously this is a truly distant future: imaging a black hole shadow would require order of ten microsecond angular resolution. However, what might be realistically foreseen is the tracking of the wobbling image centroid that a spot is supposed to produce (Hamaus et al., 2009; Zamaninasab et al., 2010). With the polarimetric resolution, the wobbling could provide an excellent evidence proving the presence of the orbiting feature. See Figures 1–4 for examples of the expected form of the spot images and the corresponding centroid tracks in a simplified case of a model spot endowed with an intrinsic polarization that remains constant in the co-orbiting frame.

Figure 1 assumes a spot rotating rigidly at constant radius near above the ISCO. Figure 2 corresponds to the case of intrinsically polarized spot radiation of which is recorded behind the polarization filter. Orientation of the filter is fixed and indicated in the top-right corner of the plot. Correspondingly, Fig. 3 shows the tracks of the image centroid. Albeit the tracks are not identical in the two orientations of the polarization filter, the difference is rather subtle. Notice that the project of detecting the centroid motion does not necessarily have to be limited to the X-ray domain. In view of recent results on Sagittarius A* flares, which have been reported in X-rays as well as in the near infrared, submillimiter and the radio spectral bands (Eckart et al., 2008), the immediate vicinity of the black hole can be probed by various techniques. The simultaneous time-dependent measurements equipped with the polarimetric resolution seem to be a final goal of this effort.

In Figure 4, the contribution from a time-evolving evolving spot and the (axially symmetric and stationary) background disc are taken into account. When put in this way, the spot represents a travelling disturbance in the disc medium, while the effect of the background disc causes a small but persisting offset of the centroid track towards the Doppler enhanced side of the disc. The changes predicted for the observed signal are now visibly larger and they are caused by the interplay between the relativistic effects and the shearing decay of the spot.

It may be worth reminding the reader that the KY code, employed in our computations, is publicly available, either as a part of the XSPEC package or directly from the authors (Dovčiak et al., 2004). The current version allows the user to include the polarimetric resolution and to compute the observational consequences of strong-gravity effects from a Kerr black hole accretion disc. Within the XSPEC notation, this polarimetric resolution is encoded by a switch defining which of the four Stokes parameters is returned in the photon count array at the moment of the output from the model evaluation. This way one can test and combine various models, and pass the resulting signal through the response matrices of different instruments.

4 CONCLUSIONS

The task of detecting the relativistic effects and in this way determining the physical parameters of the black hole systems seems to be feasible in near future. Among possible ways to reach the goal, time-dependent polarization profiles, such as those expected from orbiting spots, play an important role. In our work, the adopted approach is based on mapping the Kerr black hole equatorial plane onto the observer's plane at radial infinity. Off equatorial features are first projected onto the disc plane, hence imposing the vertically averaged approximation. A conceptually similar problem of a vertically thick oscillating torus has been studied recently by Horák and Bursa (2010), who employed a new three-dimensional code and with its help examined different modes of the torus oscillation.

To conclude, the strong gravity effects can be revealed as the observable direction of polarization is changed upon light propagation near a black hole. This may be relevant not only for the inner regions of active galactic nuclei, for which we assumed the X-ray reflection as a mechanism producing spectral and polarimetric features, but also for the radiation coming from individual blobs of gas orbiting near the Galaxy Center, where an interplay between synchrotron and inverse Compton mechanisms is expected to play a role. Spots are among viable models capable to explain the occurrence about once per day of flares from within a few milli-arcseconds of the supermassive black hole, Sagittarius A*. Because of short time-scales the flares cannot be understood in terms of viscous processes in the standard accretion disc with some appreciable accretion rate. It has been widely known that the flares from the very vicinity of the black hole are highly polarized in near-infrared, however, we are still lacking any polarimetric information on this object in X-rays.

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