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Comparison of the CDM halo and MOND models of the Magellanic Cloud motion in the field of Milky Way

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ABSTRACT
There is an explanation of the rotation curves in the periphery of spiral galaxies based on MOdified Newtonian Dynamics (MOND). Considering the motion of Magellanic Clouds in the gravitational field of Milky Way, we compare predictions of the CDM halo model with the cosmic repulsion term included to those obtained in the framework of the MOND theory. Our results demonstrate that the predictions of the CDM halo and MOND models differ very substantially, especially in the case of the Large Magellanic Cloud motion.

Keywords: MOND – galactic motion – Milky Way – Magellanic Clouds

1 INTRODUCTION

An alternative to the model of Cold Dark Matter (CDM) explanation of the rotation curves in the periphery of spiral galaxies, based on MOdified Newtonian Dynamics (MOND) (Milgrom, 1983), is realized on the Newtonian level, modifying the Newton dynamic law by introducing an additional term depending on the ratio of acceleration and some critical acceleration a_0 below which the Newton second law in not valid. The MOND dynamic law relating the acceleration a of a test particle with mass m and the acting force F takes the general form

$$m\mu(x)a = F\,, \quad x = \frac{a}{a_0}\,, \tag{1}$$

where we assume that the modification is given by the function $\mu(x)$ such that $\mu(x) \sim 1$ for $x \gg 1$ and $\mu(x) \sim x$ for $x \ll 1$. In the MOND regime the gravitational acceleration is proportional to $1/r$ and its fall is much slower in comparison with the standard Newtonian dependence $1/r^2$. The MOND is successful in explaining the rotation curves of spiral galaxies by putting $a_0 \sim 10^{-8}\mathrm{cm}\cdot\mathrm{s}^{-2}$ (Milgrom, 1983). Various interpolation formulae has been proposed to cover the transition between the Newton and MOND regime, but it seems that the simplest one that will be used later works quite well (Famaey and Binney,

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2005; Iorio, 2009). The compatibility of MOND with data from Solar System was discussed in a number of works (Sereno and Jetzer, 2006; Iorio, 2008). However, it is of high relevance to test its predictions in the case of the motion of satellite galaxies.

A relativistic covariant formulation of the MOND theory was discussed by (Bekenstein and Milgrom, 1984; Bruneton and Esposito-Farèse, 2007; Zhao, 2007; Milgrom, 2008). There are some other non-standard approaches to explanation of the galactic rotation curves without using the CDM (Iorio, 2009). Of special interest is MOdified Gravity (MOG) – a fully covariant gravity theory where a massive vector field coupled to matter exists, giving a Yukava-like modification of gravity (Moffat and Toth, 2009), but here we restrict our attention to the MOND theory.

It is of high interest to test the gravitational influence of the Milky Way on its close companions. For example, the motion of the tidal debris of the Sagittarius dwarf at 17.4 kpc from the Milky Way center was studied (Read and Moore, 2005). However, there is another important possibility for such testing due to the closest galaxies to the Milky Way, namely the Magellanic Clouds. They have their total mass much smaller than the Milky Way total mass – their mass is estimated to be smaller than $(1/10)M_{\mathrm{MW}}$. Further, their distance from the Milky Way exceeds substantially its dimension. Therefore, the Magellanic Clouds can be well approximated as test particles moving in the gravitational field of the Galaxy.

Quite recently it has been shown that the cosmic repulsion inferred from the cosmological observations (Riess et al., 2004) seems to be very important for determining the character of the satellite galaxy motion and their trajectories in the standard framework of the Galaxy model with the CDM halo (Stuchlík and Schee, 2011). The effects of the cosmological constant are on the 10 per cent level or higher, if we consider the binding mass of Milky Way relative to SMC and LMC through their initial positions and velocities. The results of the models of the motion put serious doubts on the binding of the LMC to the Milky Way if the CDM halo model is the relevant one – see also Besla et al. (2007). Nevertheless, the problem of LMC binding remains to be open due to uncertainties in determination of the initial velocity due to the Galaxy rotation velocity (Shattow and Loeb, 2009; Stuchlík and Schee, 2011). We compare here the predictions of the CDM halo model of the Galaxy gravitational field to those given by the MOND model of the satellite galaxy motion. Since the role of the cosmological constant has been shown to be important in the CDM halo model, we add the cosmic repulsion potential in the Newtonian limit

$$U_{\Lambda} = -\frac{\Lambda c^2}{6} r^2 , \tag{2}$$

to the CDM halo model – see Stuchlík and Schee (2011). On the other hand, we do not include the effects of the cosmic repulsion into the MOND model, since the trajectories predicted by the model are much closer to the Galaxy, being limited to regions where the role of the cosmic repulsion has to be suppressed. The Galaxy gravitational field is reflected by the (ellipsoidal) potential of the Galaxy disc and (spherical) potential of the Galactic bulge (Binney and Tremaine, 1987). For simplicity these can be substituted by a spherical Newtonian potential of a point source located at the Galaxy centre and having total mass of the visible Galaxy, since the motion of Magellanic Clouds is restricted to regions distant to the visible Galaxy.

2 THE GALAXY GRAVITATIONAL FIELD

The Galaxy is represented by its visible, baryonic parts, i.e. by the disk and the bulge that could be considered as central point sources, neglecting the non-sphericity of the Galaxy disc. The recent estimate of the total baryonic mass of the Galaxy is

$$M = 6.5 \times 10^{10}\, M_{\odot} \tag{3}$$

with the composition given by $M_{\rm disc} = 5 \times 10^{10}\, M_{\odot}$ and $M_{\rm bulge} = 1.5 \times 10^{10}\, M_{\odot}$ (McGaugh, 2008; Xue et al., 2008; Iorio, 2009).

The elliptical gravitational potential of the Galactic disk reads

$$U_{\rm disc} = -\frac{\xi G M_{\rm disc}}{\sqrt{x^2 + y^2 + \left(k + \sqrt{z^2 + b^2}\right)^2}}\,, \tag{4}$$

while the galactic bulge potential is simulated by

$$U_{\rm bulge} = -\frac{G M_{\rm bulge}}{r + c}\,, \tag{5}$$

where $\xi = 1$, $k = 6.5$ kpc, $b = 0.26$ kpc, $c = 0.7$ kpc. We shall compare, for completeness, the effect of the detailed potential

$$U = U_{\rm disc}(M_{\rm disc}) + U_{\rm bulge}\big(M_{M_{\rm bulge}}\big) \tag{6}$$

and the point Newtonian potential $U_{\rm PN}(M_G)$.

3 THE MOND MODEL OF GRAVITATIONAL INTERACTIONS ON COSMIC SCALES

The MOND is invented in order to enable explanation of matter motion in the outer parts of galaxies, including the Milky Way, where discrepancy between the rotation curves of matter and the gravitational effect of galactic visible matter is observed. Usually, this discrepancy is explained by the effect of an invisible CDM, while MOND is trying to explain it by modification of the Newton dynamical law (Milgrom, 1983), modifying the acceleration of matter at large distances from the galaxy center.

3.1 Modification of the Newton gravitational law

Considering the Newtonian gravitational force, the MOND dynamical law reads

$$m\mu(x)a = -G\frac{Mm}{r^2}\,, \tag{7}$$

where $\mu(x)$ is the modifying acceleration function, $x = a/a_0$ is its argument determining the magnitude of the modification and a_0 is the critical acceleration specifying the limit of validity of the standard Newtonian mechanics. From fitting of rotational curves in the

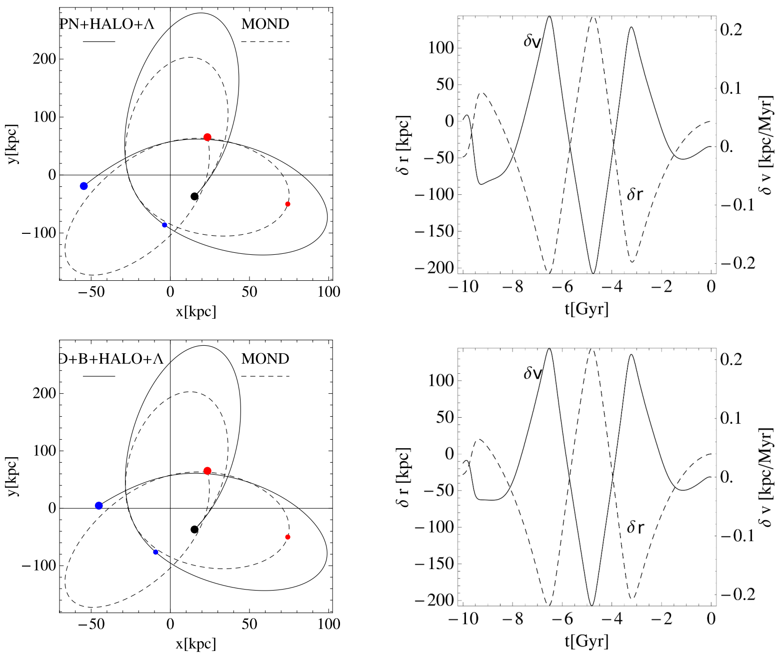

Figure 1. Comparison of MOND model with CDM models with different gravitational potentials $U_{\rm PN} + U_{\rm halo}$ (*top*), $U_{\rm disk} + U_{\rm bulge} + U_{\rm halo} - \Lambda c^2 r^2/6$ (*bottom*) is plotted. On the left the trajectories of SMC are plotted. There are three types of dots in the plot. Big black refers to the time instant $t = 0$, the small dots refer to the time instant $t = 5$ Gyr and big colored dots refer to the time instant $t = 10$ Gyr. The red color dots belong to dashed lines and blue ones to the solid lines. On the right the functions $\delta r = r_1 - r_2$ and $\delta v = v_1 - v_2$ are plotted where index 1 refers to MOND model and index 2 refers to CDM model.

Milky Way and other spiral galaxies the critical acceleration is established to be (Begeman et al., 1991)

$$a_0 = 1.2 \times 10^{-10} \mathrm{m} \cdot s^{-2} \tag{8}$$

giving thus the acceleration scale. Then in terms of the interpolation function $\mu(x)$ the actual acceleration is related to the Newtonian one by $\mathbf{a}_N = \mu(x)\mathbf{a}$ (McGaugh, 2008).

Clearly, for any gravitating mass a critical radius r_0 related to the critical acceleration can be defined by the relation

$$r_0 = \left(\frac{GM}{a_0}\right)^{1/2} \tag{9}$$

that represents a critical distance from the source of the gravitational field beyond which the MOND regime becomes effective. Using the critical value of a_0 determined by fitting the rotational curves of galaxies (8) and the total mass of the visible galactic disc and bulge

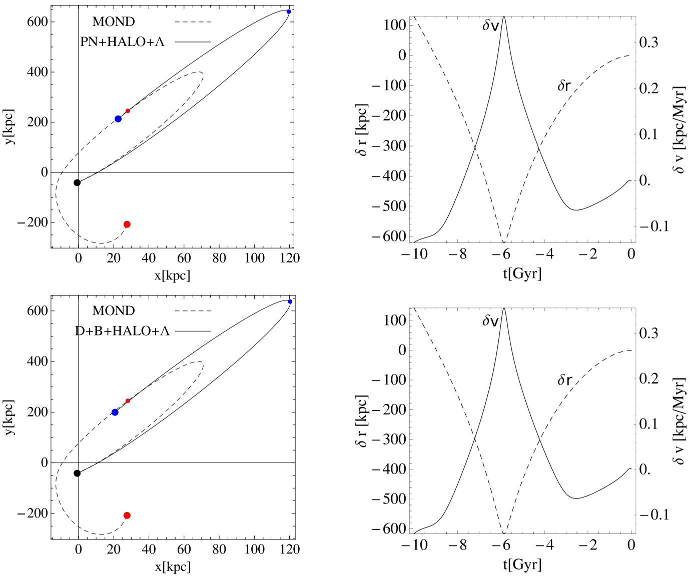

Figure 2. Comparison of MOND model with CDM models with different gravitational potentials $U_{\mathrm{PN}} + U_{\mathrm{halo}}$ (*top*), $U_{\mathrm{disk}} + U_{\mathrm{bulge}} + U_{\mathrm{halo}} - \Lambda c^2 r^2/6$ (*bottom*) is plotted. On the left the trajectories of LMC are plotted. There are three types of dots in the plot. Big black refers to the time instant $t = 0$, the small dots refer to the time instant $t = 5\,\mathrm{Gyr}$ and big colored dots refer to the time instant $t = 10\,\mathrm{Gyr}$. The red color dots belong to dashed lines and blue ones to the solid lines. On the right the functions $\delta r = r_1 - r_2$ and $\delta v = v_1 - v_2$ are plotted where index 1 refers to MOND model and index 2 refers to CDM model.

of the Milky Way ($M \sim 6.5 \times 10^{10}\,M_{\odot}$), we arrive at the characteristic radius relevant for the Milky Way

$$r_0 \sim 2.62 \times 10^{20}\,\mathrm{m} \sim 8.45\,\mathrm{kpc}\,. \tag{10}$$

representing nearly $2/3$ of the visible Galaxy extension.

3.2 The modification function and the critic acceleration

The modification function $\mu(x)$ interpolating transition between the Newtonian and fully MOND regimes was originally given in the form (Bekenstein and Milgrom, 1984)

$$\mu(x) = \frac{x}{\left(1 + x^2\right)^{1/2}}\,. \tag{11}$$

However, there is a simpler possibility (Famaey and Binney, 2005)

$$\mu(x) = \frac{x}{1 + x} \tag{12}$$

that yields better results in fitting the rotation velocity curves in the Milky Way and galaxy NGC 3198 (Zhao and Famaey, 2006; Famaey et al., 2007). The effective MOND “gravitational" acceleration can then be given by (Iorio, 2009)

$$a = \frac{a_N}{2}\left[1 + \left(1 + \frac{4a_0}{a_N}\right)^{1/2}\right]. \qquad (13)$$

Using the critical radius r_c, we can express the MOND acceleration in the form

$$a = -\frac{1}{2}\frac{GM}{r^2}\left[1 + \left(1 + \frac{4r^2}{r_0^2}\right)^{1/2}\right]. \qquad (14)$$

3.3 Modified gravitational potential and the motion of Magellanic Clouds around Milky Way

The MOND theory can be expressed by a modification of the Newtonian gravitational potential. The form of this modification is determined by the function $\mu(x)$ and using the explicit form of this function (12) we obtain the MOND gravitational potential in the form

$$\Phi_{\mathrm{MOND}} = \frac{GM}{2r} + \frac{GM}{2r}\sqrt{1 + \frac{4r^2}{r_0^2}} - \frac{GM}{r_0}\sinh^{-1}\left(\frac{2r}{r_0}\right). \qquad (15)$$

Notice that we assume spherically symmetric source of gravity neglecting thus all the details of the galactic gravitational field; of course, we do not consider the CDM halo gravitational potential. In general (non-relativistic) non-spherical situations the modified Poisson equation (Bekenstein and Milgrom, 1984)

$$\nabla \cdot \left[\mu\left(\frac{|\nabla U|}{a_0}\right)\nabla U\right] = 4\pi G\varrho \qquad (16)$$

must be used to determine the MOND potential and, consequently, acceleration. Of course, for our purposes, the gravitational acceleration given by Eq. (15) corresponding to the simplest version of MOND using the spherically symmetric acceleration formula is quite convenient. We consider the point source with $M_G = M_{\mathrm{disk}} + M_{\mathrm{bulge}}$.

4 CDM HALO MODEL

The dark matter halo is assumed spherical and its gravitational potential can be represented by the logarithmic formula of the form (Binney and Tremaine, 1987)

$$U_{\mathrm{halo}} = v_{\mathrm{halo}}^2 \ln\left(r^2 + d^2\right), \qquad (17)$$

where $v_{\mathrm{halo}} = 114\,\mathrm{km \cdot s^{-1}}$ and $d = 12\,\mathrm{kpc}$. This halo model implies the halo mass formula

$$M_{\mathrm{halo}} = \frac{2v_{\mathrm{halo}}^2 r^3}{G\left(r^2 + d^2\right)} \qquad (18)$$

giving mass of the Galaxy halo (Iorio, 2009)

$$M_{\mathrm{halo}}(r = 60\,\mathrm{kpc}) = 3.5 \times 10^{11}\, M_\odot \qquad (19)$$

in agreement with value of $M_{\rm halo}(r = 60\,{\rm kpc}) = (4.0 \pm 0.7) \times 10^{11}\, M_{\odot}$ used in (Xue et al., 2008). For different models of the CDM halo (see, e.g. Einstein, 1939; Lake, 2004; Haager, 1997, 1998; Saxton and Ferreras, 2010).

For halos more extended, crossing the radius of $r \sim 60\,{\rm kpc}$ corresponding approximately to the present positions of both the SMC and LMC, the halo mass and its influence on the motion of the Magellanic Clouds will be higher. For details see Stuchlík and Schee (2011) where the halo extension and its mass are controlled by the so called cut-off radius, assuming the same conditions to be fulfilled at the reference radius of $r \sim 60\,{\rm kpc}$. Here we adopt the results of Xue et al (Xue et al., 2008) giving the CDM halo mass $M_{\rm halo} = 1 \times 10^{12}\, M_{\odot}$. For simplicity, we do not consider here the role of the dynamical friction Mulder (1983) on the motion of the SMC and LMC through the CDM halo. Of course, the dynamic friction effect is irrelevant for the MOND model since it does not assume any halo.

5 MOTION OF MAGELLANIC CLOUDS AROUND MILKY WAY

The visible Galaxy gravitational field will be common for both the CDM and MOND models. For completeness, we use the detailed and simplified point potential of the visible Galaxy composed with the CDM halo and Λ term.

	x	y	z
x_i	15.3	-36.9	-43.3
v_i	-87± 48	-247± 42	149± 37

Table 1. Galactocentric coordinates (in kpc) and velocity components (in km·s^{-1}) of SMC ($r_0 = 58.9\,{\rm kpc}$, $v_0 = 302 \pm 52\,{\rm km/s}$).

	x	y	z
x_i	-0.8	-41.5	-26.9
v_i	-86± 12	-268± 11	252± 16

Table 2. Galactocentric coordinates (in kpc) and velocity components (in km·s^{-1}) of LMC ($r_0 = 49.5\,{\rm kpc}$, $v_0 = 378 \pm 18\,{\rm km/s}$).

When alternative explanations of galactic rotation velocity curves are considered, based on modified gravitational laws, the CDM halos are not taken into account and only the Galactic mass inferred mainly from the electromagnetic radiation emitted by the baryonic mass is considered.

In the MOND framework only the point source is considered, as the role of the detailed potential is shown to negligible. The recent motion of the Magellanic Clouds is characterized by their position and the velocity relative to the Galaxy plane that are presented in the Table 1 for the Small Magellanic Cloud (SMC) and in Table 2 for the Large Magellanic Cloud (LMC) (Iorio, 2009). These positions and velocities, given in the so called Galactocentric reference system (Shattow and Loeb, 2009) are taken as initial conditions in the integration of the motion equations giving trajectory of SMC and LMC in the field of the Galaxy.

We have confronted the trajectories of both SMC and LMC reflecting the influence of the Galaxy and its CDM Halo combined with the cosmological constant effect that were constructed and in detail discussed in Stuchlík and Schee (2011), with the trajectories obtained by the MOND – therefore, all the external field effects (e.g. those coming from the Andromeda galaxy) are considered as irrelevant. The trajectories are given in Fig. 1 for SMC and Fig. 2 for LMC. Clearly, the differences in the character of the trajectories of the Magellanic Clouds are substantial being of the same order as the extension of the trajectories and could thus serve potentially as an efficient test of the validity of the MOND models. Significant differences of the MOND and CDM halo trajectories have been found, both for the SMC and LMC galaxies. For LMC the differences are bigger than for SMC.

6 CONCLUSIONS

We compared the trajectories constructed using the models including the CDM halo and the cosmological constant to the those based on the MOND, modelling the rotation curves of visible Galaxy without necessity of the CDM halo. The results, shown in Fig. 1 for SMC and in Fig. 2 for LMC galaxy, indicate enormous differences in the predicted trajectories. In the case of SMC trajectories there is $\delta r \sim 100\,\mathrm{kpc}$ and $\delta v \sim 0.2\,\mathrm{kpc/Myr}$. In the case of the LMC trajectories the differences approach even higher values $\delta r \sim 500\,\mathrm{kpc}$ and $\delta v \sim 0.3\,\mathrm{kpc/Myr}$. On the other hand, the detailed description of the gravitational potential of the visible Galaxy is shown to be quite irrelevant for the motion of the Magellanic Clouds. We have found that the trajectories of both SMC and LMC constructed under the model of MOND differ significantly from the trajectories based on the CDM halo models. The CDM halo models were shown to be strongly dependent on the cosmic repulsion represented by the cosmological constant term Stuchlík and Schee (2011). It could be thus interesting to test the role of the cosmic repulsion even in the case of the MOND model. Nevertheless, we expect these effects to be suppressed relative to the CDM model since the closer binding of the SMC and LMC trajectories to the Galaxy.

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