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On radial U-H-E collisions between different mass particles

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ABSTRACT
We study the result of the U-H-E collisions of particles radially colliding in the strong gravity of Kerr superspinars. The colliding particles have different masses $m_1 \neq m_2$ and we quantify the outcome of such collision taking place at fixed radius $r = 1$ in the field of Kerr superspinar determined by spin parameter a under assumption of both inelastic and elastic collisions.

1 INTRODUCTION

Recently a wide interest is devoted to the so called Banados–Silk–White (BSW) process (Bañados et al., 2009) where centre of mass energy of colliding particles can be highly ultrarelativistic if they collide in vicinity of the black hole horizon (Zaslavskii, 2010; Harada et al., 2013; Tursunov et al., 2013), or in the strong gravity of naked singularity spacetimes, as those related to the Kerr superspinars (Stuchlík et al., 2011; Stuchlík and Schee, 2012, 2013; Stuchlík et al., 2014). In those processes it is usually assumed that the collisions are inelastic and the rest energy of the colliding particles is transformed into the energy of outgoing particles and photons. Here we shall consider also the possibility when the particles are scattered in an elastic process.

2 SPACETIME GEOMETRY AND EQUATIONS OF MOTION

According to String theory there exist a class of solutions interpreted as spinning object of mass M violating the general relativistic bound of spin of black holes, having $a > 1$. They are called Kerr superspinars which, in the astrophysical area of interest, can be primordial remnants of high energy phase of very early period of evolution of the Universe. It turns out that the geometry generated by Kerr superspinar is the well known Kerr geometry and its line element in Boyer–Lindquist coordinates read

$$\mathrm{d}s^2 = -\left(1 - \frac{2r}{\Sigma}\right)\mathrm{d}t^2 + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\,\mathrm{d}\theta^2 + \frac{A}{\Sigma}\sin^2\theta\,\mathrm{d}\varphi^2 - \frac{4ar\sin^2\theta}{\Sigma}\mathrm{d}t\,\mathrm{d}\varphi\,, \tag{1}$$

where is $\Delta = r^2 - 2r + a^2$, $\Sigma = r^2 + a^2\cos^2\theta$, and $A = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta$.

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It was shown that the equations of motion are separable and can be found by Hamilton–Jacobi separation process. For the motion in the equatorial plane we have the following set of equations of test particle motion

$$\Sigma \dot{r} = \pm\sqrt{R(r)}\,, \tag{2}$$
$$\Sigma \dot{\varphi} = -\left(aE - L_z\right) + \frac{a}{\Delta}P(r)\,, \tag{3}$$
$$\Sigma \dot{t} = -a\left(aE - L_z\right) + \frac{r^2 + a^2}{\Delta}P(r)\,, \tag{4}$$

where $\dot{} \equiv \mathrm{d}/\mathrm{d}w$ with w being the affine parameter and

$$P(r) = E\left(r^2 + a^2\right) - L_z a\,, \tag{5}$$
$$R(r) = P^2 - \Delta\left[m^2 r^2 + \left(L_z - aE\right)^2\right]\,. \tag{6}$$

There are two constants of motion introduced reflecting temporal and azimuthal symmetries of Kerr spacetime, they are covariant energy $E = -p_t$ and azimuthal angular momentum $L_z = p_\varphi$.

3 LOCALLY NON-ROTATING FRAMES

The collision process is studied in the frames connected with the zero-angular-momentum observers, those with $L_z = 0$. Such frame are commonly named as Locally Non-Rotating Frames (LNRF), and the corresponding tetrad reads

$$\omega^{(r)} = \left\{0, \sqrt{\frac{\Sigma}{\Delta}}, 0, 0\right\}\,, \tag{7}$$
$$\omega^{(\theta)} = \left\{0, 0, \sqrt{\Sigma}, 0\right\}\,, \tag{8}$$
$$\omega^{(t)} = \left\{\sqrt{\frac{\Delta\Sigma}{A}}, 0, 0, 0\right\}\,, \tag{9}$$
$$\omega^{(\varphi)} = \left\{-\Omega_{\mathrm{LNRF}}\sqrt{\frac{A}{\Sigma}}\sin\theta, 0, 0, \sqrt{\frac{A}{\Sigma}}\sin\theta\right\} \tag{10}$$

with the angular frequency of LNRF being

$$\Omega_{\mathrm{LNRF}} = \frac{2ar}{A}\,. \tag{11}$$

4 THE PARTICLES COLLISION

We assume the elastic collision between two particles taking place in the equatorial plane, $\theta = \pi/2$. The constant of motion $Q = 0$ and it takes place at $r_{\mathrm{c}} = 1$. We let collide radially freely falling (1) and radially freely receding (2) particles with constants of motion $E_1 = m_1, L_{z1} = 0, E_2 = m_2,$ and $L_{z2} = 0.$ The only non-zero components of 4-momentum in the LNRF frame are temporal and radial, i.e.

$$P_i^{(\mu)} = \left(P_i^{(t)}, P_i^{(r)}, 0, 0\right) , \tag{12}$$

which in particular case of our two particles reads

$$P_1^{(\mu)} = \left(m_1\gamma, m_1\gamma v^{(r)}, 0, 0\right) , \tag{13}$$

$$P_2^{(\mu)} = \left(m_2\gamma, -m_2\gamma v^{(r)}, 0, 0\right) . \tag{14}$$

with the radial 3-velocity component $v^{(r)}$ given by relation

$$v^{(r)} = \frac{\omega_\mu^{(r)} U^\mu}{\omega_\mu^{(t)} U^\mu} = \pm\sqrt{\frac{2\left(1+a^2\right)}{\left(1+a^2\right)^2 - \left(1-a^2\right)a^2}} . \tag{15}$$

and $\gamma = (1 - [v^{(r)}]^2)^{-1/2}$.

Just before the collision the total 4-momentum $P^{(\mu)}$ is

$$P^{(\mu)} = P_1^{(\mu)} + P_2^{(\mu)} = \left((m_1+m_2)\gamma, (m_1-m_2)\gamma v^{(r)}, 0, 0\right) . \tag{16}$$

We first assume that masses of particles remain the same after collision, then the corresponding components of 4-momenta of colliding particles after collision follow from conservation principles and from normalization of 4-momentum, i.e.

$$P'^{(t)} = P'^{(t)}_1 + P'^{(t)}_2 = P^{(t)} = (m_1+m_2)\gamma , \tag{17}$$

$$P'^{(r)} = P'^{(r)}_1 + P'^{(r)}_2 = P^{(r)} = (m_1-m_2)\gamma v^{(r)} , \tag{18}$$

$$-m_1^2 = -\left[P'^{(t)}_1\right]^2 + \left[P'^{(r)}_1\right]^2 , \tag{19}$$

$$-m_2^2 = -\left[P'^{(t)}_2\right]^2 + \left[P'^{(r)}_2\right]^2 . \tag{20}$$

Solving this set of equations and using

$$v'^{(r)}_i = \frac{P'^{(r)}_i}{P'^{(t)}_i} , \quad i = 1, 2 \tag{21}$$

the resulting radial 3-velocity of particles after collision read

$$v'^{(r)}_1 = \frac{BD + \sqrt{B^2D^2 - \left(A^2-B^2\right)\left(4A^2m_1^2 - D^2\right)}}{AD - \sqrt{A^2D^2 - \left(A^2-B^2\right)\left(4B^2m_1^2 + D^2\right)}} , \tag{22}$$

$$v'^{(r)}_2 = \frac{2B\left(A^2-B^2\right) - BD - \sqrt{B^2D^2 - \left(A^2-B^2\right)\left(4A^2m_1^2 - D^2\right)}}{2A\left(A^2-B^2\right) - AD + \sqrt{A^2D^2 - \left(A^2-B^2\right)\left(4B^2m_1^2 + D^2\right)}} , \tag{23}$$

where we have introduced $A = (m_1 + m_2)\gamma$, $B = (m_1 - m_2)\gamma v^{(r)}$, and

$$D = (m_1 + m_2)\left[m_1 - m_2 + (m_1 + m_2)\gamma^2\right] - (m_1 - m_2)^2\gamma^2\left[v^{(r)}\right]^2 . \tag{24}$$

In the second case we assume that the mass of collision products are the same having value of m we have

$$-m^2 = -\left[P'^{(t)}_1\right]^2 + \left[P'^{(r)}_1\right]^2 , \tag{25}$$

$$-m^2 = -\left[P'^{(t)}_2\right]^2 + \left[P'^{(r)}_2\right]^2 . \tag{26}$$

From Equations (17), (18), (25), and (26) the resulting radial 3-velocities of collision products are

$$v'^{(r)}_{1\pm} = \frac{B \pm A\sqrt{1 - 4m^2}}{A \mp \sqrt{2A^2 - B^2\left(1 + 4m^2\right)}} , \tag{27}$$

$$v'^{(r)}_{2\pm} = \frac{2A - B \mp A\sqrt{1 - 4m^2}}{2B - A \pm \sqrt{2A^2 - B^2\left(1 + 4m^2\right)}} . \tag{28}$$

In the third case we asked a question, what are the conditions for masses of colliding particles and the masses of the products if we want the products of the collision to became static just after the collision? In this case we have following set of equations

$$P'^{(t)}_1 + P'^{(t)}_2 = \gamma(m_1 + m_2) , \tag{29}$$

$$\underbrace{P'^{(r)}_1}_{0} + \underbrace{P'^{(r)}_2}_{0} = 0 = \gamma v^{(r)}(m_1 - m_2) . \tag{30}$$

Which imply the masses of particles before collision are same $m_1 = m_2$ and the masses of the products is determined by formula

$$m = \gamma m_1 = \frac{1}{\sqrt{1 - [v^{(r)}]^2}} m_1 . \tag{31}$$

The characteristic parameter of collision is the centre-of-mass energy $E_{\rm CM}$. It is the total energy of system measured by observer at rest in CM. In the case of two particle collision we have $P_{\rm tot} = P_1 + P_2$ which imply the energy

$$E^2_{\rm CM} = -P_{\rm tot} \cdot P_{\rm tot} = m_1^2 + m_2^2 - 2g_{\mu\nu}P_1^{\mu}P_2^{\nu} , \tag{32}$$

and, in our particular case, it reads

$$E^2_{\rm CM} = m_1^2 + m_2^2 + \frac{2}{r^2}\left\{\left[m_1 m_2\left(r^2 + a^2\right)^2 + 2r\sqrt{m_1 m_2}\left(r^2 + a^2\right)\right]\frac{1}{\Delta} - a^2 m_1 m_2\right\} . \tag{33}$$

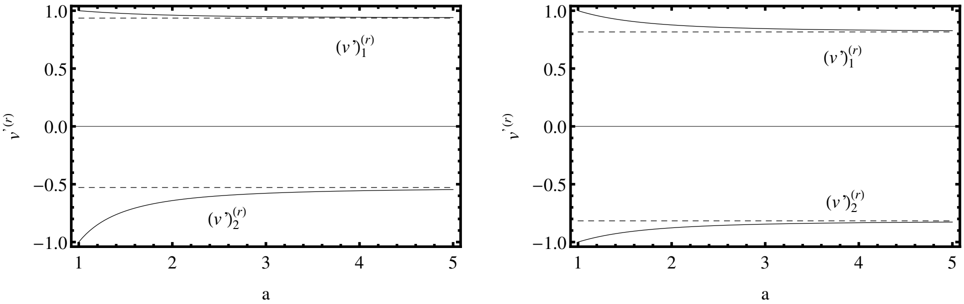

Figure 1. Plots of $v'^{(r)}_1$ and $v'^{(r)}_2$ curves as functions of spin parameter for fixed values of particle masses. Plots on the left (right) are constructed for $m_1 = 2$ and $m_2 = 1$ ($m_1 = 1$ and $m_2 = 2$).

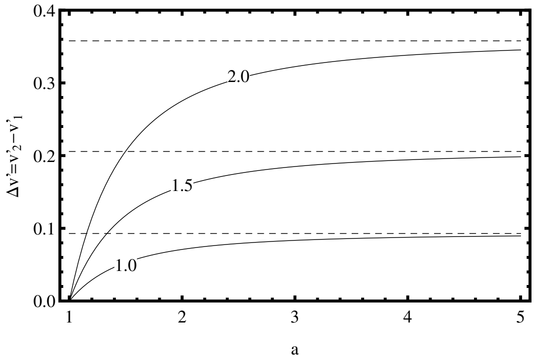

Figure 2. The difference between the magnitude of velocities of two particles collision products gaining after it same mass m. Each curve is plotted for a representative value of collision product masses. Each curve is asymptotically for $a \to \infty$ getting to limiting value which in presented cases are $\Delta v'_{\rm lim}(m = 1.0) = 0.0928676$, $\Delta v'_{\rm lim}(m = 1.5) = 0.205702$, and 0.357901.

5 RESULTS

We let collide two particles with $L_z = 0$ at $r = 1$. The particles have distinct masses $m_1 \neq m_2$. With respect to collision products masses we have studied two situations:

- Masses of products do not change during collision. We first study the case of $m_1 = m'_1 = 1$ and $m_2 = m'_2 = 2$ and of $m_1 = m'_1 = 2$ and $m_2 = m'_2 = 1$.
- The masses of products is the same $m'_1 = m'_2 = m$. In our simulations the mass $m = 1$, 1.5, and 2.0.

The outcome of the collision is reflected in the plots of curves $v'^{(r)}_1(a)$ and $v'^{(r)}_2(a)$ in Fig. 1. There are two limiting values as spin $a \to \infty$, $v'^{(r)}_{1\,\rm limit} = -0.528321$ and $v'^{(r)}_{2\,\rm limit} = 0.935984$ in the first choice of particle masses and $v'^{(r)}_{1\,\rm limit} = \pm 0.81651$ for the second choice of particle masses. The maximal values of velocities of particles is reached for spins close to extreme Kerr black hole state.

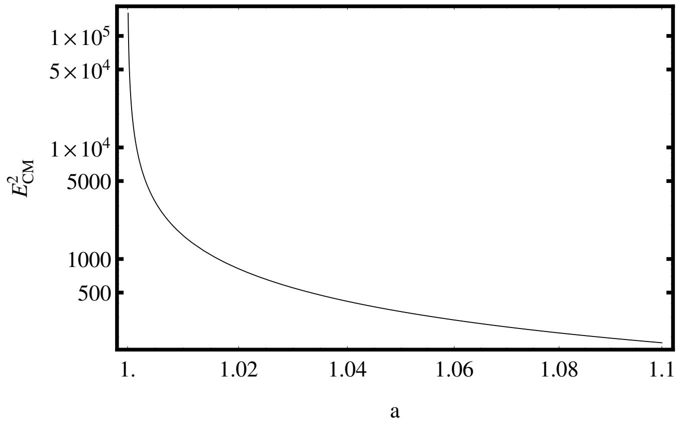

Figure 3. We demonstrate the strength of head on collision of two radially moving test particles with masses $m_1 = 1$ and $m_2 = 2$, which are moving radially, by the square of centre-of-mass energy E^2_{CM}.

The square of centre of mass energy E^2_{CM}, given by formula (33), of collision taking place at $r = 1$ of two radially moving particles with masses $m_1 = 1$ and $m_2 = 2$ is given at Fig. 3.

6 CONCLUSION

We can conclude that in the case of the elastic collisions, the efficiency is largest for near-extreme Kerr superspinars, similarly to the case of the collisions where the rest energy of the colliding particles is transformed into energy of outgoing particles and photons.

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