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String loop oscillation model applied to the twin HF QPOs in the atoll source 4U 1636-53

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ABSTRACT
The current-carrying string loops oscillating around a stable equilibrium position in the Kerr background are considered to explain the twin high-frequency quasiperiodic oscillations (HF QPOs) observed in the low-mass X-ray binary 4U 1636-53 containing a neutron star. The frequencies of the radial and vertical string loop oscillations are governed by the mass and spin parameters of the neutron star, and by the string parameter describing combined effects of its tension and angular momentum. The frequencies of the radial and vertical modes of the string loop oscillations can cover the large scatter of the twin HF QPO data observed in the 4U 1636-53 source, but the estimates of the mass M and spin a of the neutron star are rather high, $M \sim 2.65\, M_{\odot}$ and $a \sim 0.45$, while related to the theory of the neutron star structure. Therefore, the string loop oscillation model in the case of the 4U 1636-53 source requires a correction based on an electrically charged string loop interacting with the magnetic field of the neutron star.

Keywords: string loop oscillations – X-ray variability – HF QPO observations

1 INTRODUCTION

Current-carrying string loops can represent combined systems of magnetic field and plasma, exhibiting a string-like behaviour due to dynamics of the magnetic field lines (Semenov et al., 2004; Christensson and Hindmarsh, 1999), or due to the thin flux tubes of magnetized plasma simply described as 1D strings (Semenov and Bernikov, 1991; Cremaschini and Stuchlík, 2013; Cremaschini et al., 2013; Kovář, 2013). The string loops are governed by their tension and angular momentum. (Recall that first the cosmic strings were introduced as remnants of the phase transitions in the very early universe (Vilenkin and Shellard, 1995), and strings represented as superconducting vortices were introduced by (Witten, 1985)). Dynamics of axially symmetric string loops in axially symmetric backgrounds is relatively very simple and can be fully governed by an appropriately defined effective potential, in analogy with test particle motion (Jacobson and Sotiriou, 2009; Kološ and Stuchlík, 2010; Stuchlík and Kološ, 2012a; Kološ and Stuchlík, 2013). The current-carrying string loops

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moving axisymmetrically along the symmetry axis of the Kerr or Schwarzschild–de Sitter black holes can be relevant in astrophysical processes (Jacobson and Sotiriou, 2009; Kološ and Stuchlík, 2010; Stuchlík and Kološ, 2012a; Kološ and Stuchlík, 2013). Electrically charged current-carrying string loops oscillating in combined external gravitational and electromagnetic fields can be also fully described by an effective potential, if the string loop and the background have common axial symmetry (Tursunov et al., 2013).

Recently, the axisymmetric current-carrying string loops were considered as a model of ultrarelativistic jet formation due to transmutation effect governing transmission of the internal energy of the oscillatory motion to the energy of translational motion (Stuchlík and Kološ, 2012a,b; Kološ and Stuchlík, 2013). On the other hand, it has been shown that small oscillations of the current-carrying string loops can explain frequency of the HF QPOs observed with frequency ratio 3:2 in the three Galactic microquasars, GRS 1915+105, XTE 1550-564, GRO 1655-40, i.e. Low-Mass X-ray Binary (LMXB) systems containing a black hole (Stuchlík and Kološ, 2014b), and the special set of HF QPOs observed in the peculiar neutron star low-mass X-ray binary XTE 1701-407 (Stuchlík and Kološ, 2014a). The string loop oscillation model in both the cases assumes relevance of some resonant phenomena and predicts reasonable restrictions on the values of the mass and spin of the black holes in the microquasars, and the neutron star in the XTE J1701-407 source.

Here we test the string loop oscillation model in the case of the well studied atoll source 4U 1636-53, where a large scatter of the twin HF QPOs is observed (Barret et al., 2005, 2006; Belloni et al., 2007a; Wang et al., 2013, 2014). Then a different approach has to be applied in order to match the observed twin HF QPOs where resonant phenomena are not taken into consideration. On the other hand, we keep the assumption that mass of the neutron star is large enough so that the external field of the neutron star can be well approximated by the Kerr geometry (Urbanec et al., 2013).

2 STRING LOOP OSCILLATION MODEL

2.1 Motion of axisymmetric string loops

Dynamics of an axisymmetric current-carrying string loop in a given axially symmetric and stationary, Kerr, spacetime with metric $g_{\alpha\beta}$ was treated in (Jacobson and Sotiriou, 2009; Kološ and Stuchlík, 2013). Oscillations of such string loops have been studied in (Stuchlík and Kološ, 2014b). The oscillations of the string loop can be characterized by two parameters, J and ω, reflecting the effect of the magnitude and components of the angular momentum and the string tension (Stuchlík and Kološ, 2014b).

As demonstrated in (Larsen, 1993; Stuchlík and Kološ, 2014b), the string loop motion can be described by the Hamilton equations and an appropriately defined Hamiltonian H with a dynamic, H_{D}, and a potential, H_{P}, parts. The potential part is related to the constants of motion associated to the background symmetries, namely, the energy E and the angular momentum parameters J and ω. The boundary of the string loop motion is given by vanishing of the potential parts of the Hamiltonian that implies the so called energy boundary function $E_{\mathrm{b}}(r, \theta; a, J, \omega)$, (Kološ and Stuchlík, 2013; Stuchlík and Kološ, 2014b), serving as an effective potential of the string loop motion. The turning points of the string loop motion are determined by the condition $E = E_{\mathrm{b}}(r, \theta; a, J, \omega)$.

In the Kerr metric and the standard Boyer–Lindquist r, θ coordinates, (Carter, 1973), the energy boundary function takes the form (Stuchlík and Kološ, 2014b),

$$E_{\mathrm{b}}(r,\theta;a,J,\omega)=\frac{4a\omega J^2 r}{\left(\omega^2+1\right)G}+\sqrt{\Delta}\left(\frac{J^2R^2}{G\sin(\theta)}+\sin(\theta)\right), \tag{1}$$

where

$$G(r,\theta;a)=\left(a^2+r^2\right)R^2+2a^2r\sin^2(\theta), \tag{2}$$

and

$$R^2=r^2+a^2\cos^2\theta, \quad \Delta=r^2-2Mr+a^2, \tag{3}$$

a denotes spin and M mass parameters of the Kerr spacetimes. Here we consider only the Kerr black hole spacetimes ($a<M$), at the external region located above the outer event horizon given by

$$r_+=M+\left(M^2-a^2\right)^{1/2}. \tag{4}$$

Of course, for the exterior of neutron stars we have to consider only the part of the Kerr spacetime limited by the condition $r\geq R_{\mathrm{surface}}>r_+$.

In the following, we shall use for simplicity the dimensionless radial coordinate $r\to r/M$, dimensionless time coordinate $t\to t/M$ and dimensionless spin $a\to a/M$; this is equivalent to using of $M=1$ in the metric tensor. We will return to the dimensional quantities in the Section 3.

Detailed discussion of the properties of the energy boundary function $E_b(r,\theta)$ is presented in (Kološ and Stuchlík, 2013) for both the Kerr black hole and naked singularity spacetimes. Here we focus on the properties in the black hole spacetimes that can be relevant for rotating neutron stars as demonstrated in (Urbanec et al., 2013; Török et al., 2008) – in this case the local extrema of the energy boundary function can be located in the equatorial plane only.

The local extrema of the energy boundary function $E_{\mathrm{b}}(r;a,J,\omega)$, governing the equilibrium positions of the string loops in the equatorial plane ($\theta=\pi/2$), are determined by the relation (Kološ and Stuchlík, 2013; Stuchlík and Kološ, 2014b),

$$J^2=J_{\mathrm{E}}^2(r;a,\omega)\equiv\frac{(r-1)\left(\omega^2+1\right)H^2}{4a\omega\sqrt{\Delta}\left(a^2+3r^2\right)+\left(\omega^2+1\right)F}, \tag{5}$$

where

$$H(r;a)=r^3+a^2(2+r), \quad F(r;a)=(r-3)r^4-2a^4+a^2r\left(r^2-3r+6\right). \tag{6}$$

The oscillations of the string loops around a stable equilibrium position in the Kerr background has been discussed in (Kološ and Stuchlík, 2013; Stuchlík and Kološ, 2014b). In the basic approximation, for the first term in the perturbation expansion of the Hamiltonian around the stable equilibrium positions, the oscillations can be separated to two modes of

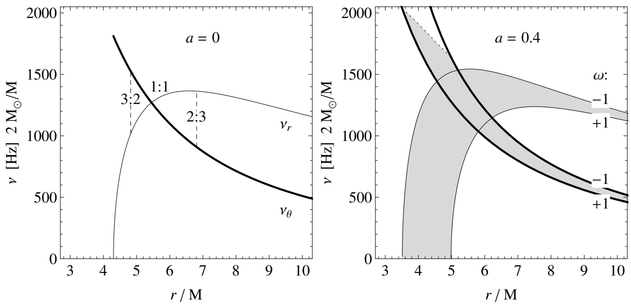

Figure 1. String-loop oscillatory frequencies ν_r (thin curves) and ν_θ (thick curves), calculated for the Kerr metrics with $M = 2\,M_\odot$. Their radial profiles are illustrated for values of dimensionless spin $a = 0, 0.4$ that are characteristic of our study of neutron star system. We demonstrate extension of the frequency radial profiles for the complete range of the string loop parameter $\omega \in \langle -1, 1 \rangle$. The vertical frequency curves are restricted to the region of existence (zero point) of the corresponding radial frequency curves – the relevant region is greyed.

independent linear-harmonic oscillations in the radial and vertical direction. The higher order terms of the expansion govern subsequent transition to the quasiharmonic oscillations and finally to the fully chaotic oscillatory motion; it is very important that frequency of the quasiharmonic oscillations of the string loops agrees with frequency of their harmonic oscillations, (Stuchlík and Kološ, 2014b).

The rotating neutron stars are conveniently described by the Hartle–Thorne geometry that is characterized in the exterior part by three parameters: mass M, dimensionless spin a, and dimensionless quadrupole moment q, as shown in (Hartle, 1967; Hartle and Thorne, 1968; Chandrasekhar and Miller, 1974). The Hartle–Thorne model can be used for rotating neutron stars with rotation frequency significantly smaller that the mass-shedding frequency, $f_{m-sh} \sim 1100$ Hz, and spin $a < 0.5$ (Urbanec et al., 2013). The rotation frequency of neutron stars described well by the Hartle–Thorne model can be as high as $f_\mathrm{rot} \sim 600$ Hz.

For $q/a^2 = 1$, the Hartle–Thorne geometry coincides with Kerr geometry, and for $1 < q/a^2 < 2$ these two geometries are very close each other giving very close predictions for astrophysical phenomena (Török et al., 2008; Török et al., 2010; Bejger et al., 2010; Bini et al., 2013). It has been demonstrated recently that for a wide variety of equations of state, the Hartle–Thorne models predict $q/a^2 < 2$, if the neutron star mass is close to the maximum allowed by a given equation of state, implying thus applicability of the Kerr geometry (Urbanec et al., 2013).

2.2 Frequency of the radial and vertical harmonic oscillatory modes

For the string loop harmonic oscillations around a stable equilibrium position at a given r_0 and $\theta_0 = \pi/2$ the locally measured angular frequencies of the radial and vertical oscillatory motion reads (Stuchlík and Kološ, 2014b)

$$\omega_\mathrm{r}^2 = \frac{1}{g_{rr}} \frac{\partial^2 H_\mathrm{P}}{\partial r^2}\,, \quad \omega_\theta^2 = \frac{1}{g_{\theta\theta}} \frac{\partial^2 H_\mathrm{P}}{\partial \theta^2}\,. \tag{7}$$

The partial derivatives of the potential part of the Hamiltonian are calculated at the local minimum of the energy boundary function at r_0 and $\theta_0 = \pi/2$. The location of the stable equilibrium point is determined by the angular momentum parameters J and ω of the string loop – see (Stuchlík and Kološ, 2014b).

The locally measured angular frequencies are connected to the angular frequencies related to distant observers, $\Omega_{(\mathrm{r},\theta)}$, by the gravitational redshift transformation (Stuchlík and Kološ, 2014b),

$$\Omega_{(\mathrm{r},\theta)} = \frac{\omega_{(\mathrm{r},\theta)}}{P^t} \,. \tag{8}$$

If the angular frequencies $\Omega_{(\mathrm{r},\theta)}$, or frequencies $\nu_{(\mathrm{r},\theta)}$, are expressed in the physical units, their dimensionless form has to be extended by the factor c^3/GM. Then the frequencies of the string loop oscillations measured by the distant observers are given by

$$\nu_{(\mathrm{r},\theta)} = \frac{1}{2\pi}\frac{c^3}{GM}\Omega_{(r,\theta)} \,. \tag{9}$$

The same factor occurs in the case of the orbital and epicyclic frequencies of the geodesic motion in the Kerr spacetime (Aliev and Galtsov, 1981; Török and Stuchlík, 2005; Stuchlík and Schee, 2012). The mass-scaling of the frequencies of the radial and vertical oscillations is the same for both the current-carrying string loops and test particles and we are approved to expect that the string loop oscillations could serve as an explanation of the HF QPOs observed in the strong gravity regions of black holes and neutron stars. In the Kerr geometry, the angular frequencies of the string loop oscillations related to distant observers take the dimensionless form (Stuchlík and Kološ, 2014b),

$$\Omega_{\mathrm{r}}^2(r; a, \omega) = \frac{J_{\mathrm{E(ex)}}\Big(2a\omega\sqrt{\Delta}\big(a^2+3r^2\big) + \big(\omega^2+1\big)F_1\Big)}{2r\big(a^2(r+2)+r^3\big)^2 F_3^2} \,, \tag{10}$$

$$\Omega_{\theta}^2(r; a, \omega) = \frac{\sqrt{\Delta}\Big(2a\omega\sqrt{\Delta}\big(2a^2-3a^2r-3r^3\big) + \big(\omega^2+1\big)F_2\Big)}{r^2\big(a^2(r+2)+r^3\big)F_3} \,, \tag{11}$$

where

$$F_1(r, a) = a^2r^3 - a^2\Delta + r^5 - 2r^4 \,, \tag{12}$$

$$F_2(r; a) = a^4(3r-2) + 2a^2(2r-3)r^2 + r^5 \,, \tag{13}$$

$$F_3(r; a, \omega) = 2a\omega\big(a^2+3r^2\big) + \sqrt{\Delta}\big(\omega^2+1\big)\big(r^3-a^2\big) \,, \tag{14}$$

$$\begin{aligned} J_{\mathrm{E(ex)}}(r; a, \omega) \equiv{} & \big(\omega^2+1\big)H(r-1)\big(6a^2r - 3a^2r^2 - 6a^2 - 5r^4 + 12r^3\big) \\ & + 4a\omega H\Delta^{-1/2}\Big[\big(a^2+3r^2\big)\Big(\Delta - \big(r-1\big)^2\Big) - 6\Delta r(r-1)\Big] \\ & - \big(\omega^2+1\big)\Big[FH + 2F\big(a^2+3r^2\big)(r-1)\Big] \\ & + 8a\omega\sqrt{\Delta}\big(a^2+3r^2\big)^2(r-1) \,. \end{aligned} \tag{15}$$

The function $J_{\mathrm{E(ex)}}(r; a, \omega)$ determines the local extrema of the function $J_{\mathrm{E}}(r; a, \omega)$ and character of the local extrema of the energy boundary function. Its zero points correspond to the marginally stable equilibrium positions of the string loops – the frequency of the radial oscillatory modes of the string loops vanishes there. The conditions

$$J_{\mathrm{E(ex)}} = 0 \quad \text{and} \quad J_{\mathrm{E}}^2 \geq 0\,, \tag{16}$$

satisfied simultaneously, put the limit on validity of the formulae giving the angular frequencies of the radial and vertical oscillations (Stuchlík and Kološ, 2014b).

In the case of spherically symmetric spacetimes, $a = 0$, the parameter ω is irrelevant for the string loop oscillatory motion. The vertical oscillations are then fully governed by the gravity effect of the black hole (or neutron star) and the string tension and angular momentum play no role (Kološ and Stuchlík, 2013). The frequency of the vertical oscillations of the string loops equals those of the test particle epicyclic motion. In the rotating Kerr spacetimes, even for string loops with $\omega = 0$, the vertical harmonic oscillations are different for the string loops and test particles, implying relevance of the string tension and angular momentum even in the simplest state of $\omega = 0$ (Stuchlík and Kološ, 2014b).

Behaviour of the radial profiles of the radial and vertical frequencies of the string loop harmonic oscillations is illustrated in Figure 1 for two characteristic values of the Kerr spin parameter $a = 0, 0.4$. In the Schwarzschild spacetime ($a = 0$), a degeneration occurs, and both the frequencies are independent of the parameter ω. In the Kerr spacetimes, the range of the radial and vertical frequencies depends on the string-loop parameter ω, and the spin parameter a of the spacetime. Extension of the range of allowed frequencies increases with increasing spin a, if we consider the full range of the angular momentum parameter ω. For all values of the spin, and at each radius where the two oscillatory modes can occur, the vertical frequency has its maximum (minimum) for string loops with $\omega = -1$ ($\omega = +1$), while the radial frequency has its maximum (minimum) for string loops with $\omega = +1$ ($\omega = -1$) – see Fig. 1.

3 TWIN HF QPOS IN ATOLL AND Z SOURCES

The low mass X-ray binary (LMXB) systems containing neutron stars are separated into two categories – the so called atoll and Z sources. This categorisation reflects distinct spectral properties of the sources and its details can be found in (Hasinger and van der Klis, 1989). Distinctions of these two classes of the neutron star LMXB systems are discussed, e.g. in (van der Klis, 2006); remarkably, the Z sources are persistent, brighter and harder than the atoll sources. Both atoll and Z sources demonstrate twin HF QPOs. Details of observed HF QPO are presented in original papers related to individual sources, a review detailed study comparing properties of HF QPOs in a large variety of observed atoll and Z sources is prepared in (Török et al., 2014), other details are presented in (Barret et al., 2005; Belloni et al., 2007b; Wang et al., 2013).

In the LMXB systems the frequencies of observed quasiperiodic oscillations range from $\sim 10^{-2}$ Hz up to $\sim 10^{3}$ Hz. We restrict attention to the kHz (high-frequency) QPOs, with frequencies in the range 200–1300 Hz that are comparable to the frequencies of the orbital motion in strong gravity near neutron stars and stellar-mass black holes (van der Klis, 2006). In the neutron-star sources, HF QPOs usually occur as two simultaneously observed peaks in the X-ray flux, with frequencies that substantially change over time – see,

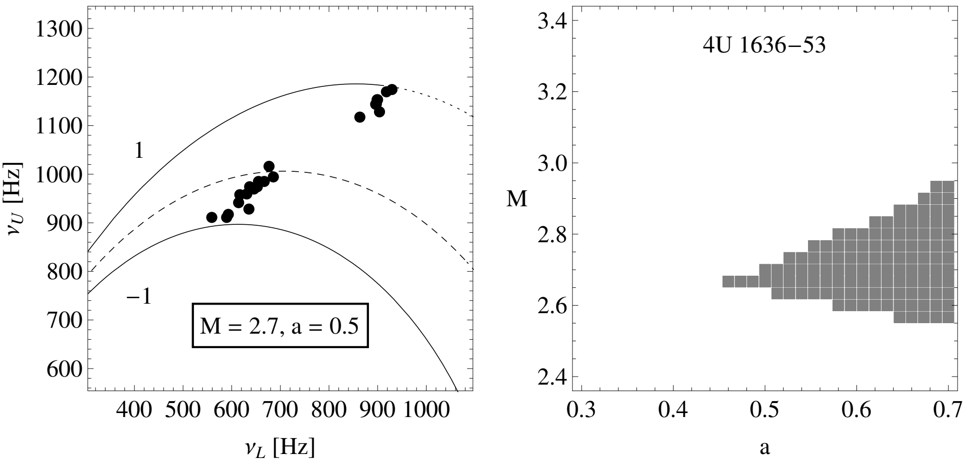

Figure 2. Restrictions on the mass M and spin a parameters of the neutron star in the 4U 1636-53 source implied by the string loop oscillation model applied to the whole variety of observational events of HF QPOs at the source. The limiting $\nu_r - \nu_\theta$ (corresponding to $\nu_{\mathrm{U}} - \nu_{\mathrm{L}}$) dependences are given for the limiting values of $\omega = \pm 1$ (*left figure*) in the case of properly chosen spacetime parameters M, a. We then give the allowed range in the space of the spacetime parameters $M - a$ where covering of all the observational twin HF QPO data in the source 4U 1636-53 is possible (*right figure*).

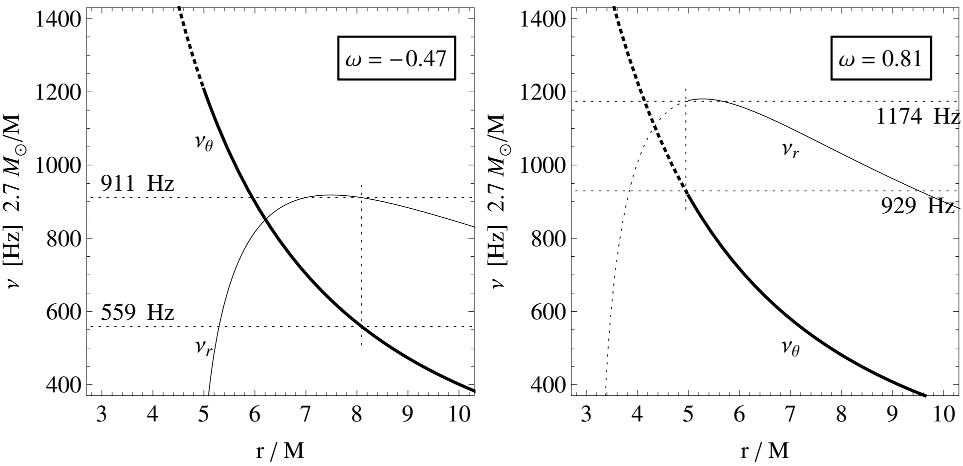

Figure 3. Examples of the radial profiles of the string-loop oscillatory frequencies ν_{r} (*thin curves*) and ν_θ (*thick curves*) as related to the two observational events, with maximal and minimal frequency ν_{U}. The parameters of the Kerr metric are mass $M = 2.7\,M_\odot$ and spin $a = 0.5$. The curved are dashed for $r < 5$, which is consider to be the inner parts of central object. The related values of the parameter ω are depicted in both subfigures, relevant frequencies are also given.

e.g. (Barret et al., 2005; Méndez, 2006).[1] According to the standard convention, we call the two peaks corresponding to a twin HF QPOs the lower and upper QPO denoting their frequencies $\nu_{\rm L} < \nu_{\rm U}$. The observed twin HF QPOs span in the atoll or Z sources a large frequency range following an approximately linear $\nu_{\rm L} - \nu_{\rm U}$ relation (Belloni et al., 2005). The frequency ratio in twin HF QPOs changes in the range of 3:2 to 5:4 in the atoll sources and in some of the Z sources, but for a variety of the Z sources the frequency ratio starts at 3:1 and finishes at 3:2. Recall that we know also a peculiar neutron star source XTE J1701-407 where the observed HF QPOs resemble those observed in the microquasars, displaying two twin HF QPOs with frequency ratio 3:2 and a single HF QPO (Pawar et al., 2013). A similar behaviour occurs in the case of the peculiar source XTE J1701-462 (Homan et al., 2007).

The string loop oscillation model has been successfully applied to explain the frequencies of the HF QPOs with ratio 3:2 observed in the three Galactic microquasars GRS 1915+105, XTE 1550-564, GRO 1655-40 (Stuchlík and Kološ, 2014b), and the frequencies of HF QPOs observed in the peculiar source XTE 1701-407 where one observed frequency is mixed with two twin frequencies with ratio very close to the 3:2 ratio, typically observed in the black hole systems (microquasars), (Stuchlík and Kološ, 2014a). In both the cases, relevance of resonant phenomena in the string loop oscillation model has been assumed, and the limits of the string loop model implied for the black hole (neutron star) mass and spin are in agreement with independent measurements of these spacetime parameters. The resonance phenomena can be relevant for the behaviour of the oscillating string loops as indicated by the Kolmogorov–Arnold–Moser (KAM) theory (Möser, 1962; Stuchlík and Kološ, 2014b), or some resonance phenomena could be relevant even for creation of the string loops, selecting thus some special radii related to the resonant phenomena.

4 FITTING THE FREQUENCIES OF THE TWIN KHZ QPOS IN THE ATOLL SOURCE 4U 1636-53

Here we concentrate our attention on the frequency distribution of the observed twin HF QPOs in the widely studied atoll source 4U 1636-53. In this case the resonant phenomena cannot be relevant in explaining the observed twin HF QPOs because of the large scatter of the 4U 1636-53 observational data spanning the whole interval of the frequency ratio 3:2–5:4 (Barret et al., 2005; Török, 2009). The data reflecting all the observed twin HF QPOs in 4U 1636-53 are illustrated in Fig. 2.

For a given twin HF QPOs observed in a given source, we have to consider fixed values of the string parameter ω and the spacetime parameters M and a. For a variety of twin HF QPOs being observed in the source, the spin and mass parameters have to be fixed, but the string loop parameter ω can be varied. Various twin frequency observations could be generated by different string loops being created and decayed successively with different values of the parameter ω reflecting locally different conditions in the source. The

[1] In the black hole systems (microquasars), the HF QPOs are detected at constant frequencies that are characteristic of a given source (Remillard and McClintock, 2006). When two or more frequencies are detected, they occur with a fixed small-number ratio; for twin observations the ratio 3:2 typically occurs (Török et al., 2005).

string-loop oscillation model thus naturally introduces a possibility of significant scatter in distribution of frequencies of the twin HF QPOs. A large scatter in the distribution of twin HF QPOs in the $\nu_L - \nu_U$ diagram can occur, if string loops with differing parameter ω arise at a fixed radius of the disc under evolving conditions, or if they arise at different radii of the disc under differing local conditions. Naturally, we can expect mix of these two possibilities. On the other hand, a regular distribution of the twin HF QPOs along a line in the $\nu_L - \nu_U$ diagram is possible only if string loops with a fixed parameter ω occur on different radii of the Kerr spacetime describing the exterior of the neutron star; however, this is not the case of the 4U 1636-53 source.

For the largely scattered twin HF QPOs in the atoll sources 4U 1636-53, the resonance phenomena are evidently irrelevant in the framework of the string loop oscillation model and will not be considered here. Each point representing a twin HF QPO in the $\nu_L - \nu_U$ diagram determines both the upper and lower frequencies, and their ratio. The frequencies given by an observed twin HF QPOs can be related to the frequencies of the radial and vertical oscillatory modes of the string loop. The upper frequency can be identified to the vertical (radial) frequency, if the oscillating string loop is located under (above) the radius of coincidence of the radial and vertical frequencies of a given string loop governed by its parameter ω.

The procedure of fitting the string loop oscillation frequencies to the observed frequencies in an observational event of a twin HF QPO has been determined in (Stuchlík and Kološ, 2014b). The fitting procedure gives for each of the observed events an allowed region of the parameter space of the spacetime parameters M, a, determined by the limiting values of the string loop parameter $\omega \in \langle -1, 1\rangle$. The resulting restriction of the spacetime and string parameters M, a, ω is then given by the conjunction of the restrictions given for individual twin-frequency data points. While this procedure works quite efficiently for the simple situations of the observed HF QPOs in the microquasars and the XTE J1701-407 neutron star source, for the complex data sets related to the atoll source 4U 1636-53 it is complex and inconvenient.

Therefore, we use a different method, giving directly the dependence of the frequency of the radial and vertical oscillatory modes in the $\nu_L - \nu_U$ diagram of the observed data points, assuming that the whole interval of the string loop parameter $\omega \in \langle -1, 1\rangle$ is relevant. For given mass parameter M, the fitting predicts only one line of the radial and vertical frequencies for the spin $a = 0$, due to the degeneracy of the radial profiles of the string loop oscillation frequencies in the Schwarzschild spacetimes ($a = 0$), i.e. their independence of the stringy parameter ω. Extension of the frequency region covered by the lines of the radial and vertical frequencies being related to the whole interval of the string loop parameter $\omega \in \langle -1, 1\rangle$ (i.e. the interval of allowed values of frequencies) increases with increasing spin a. We have to look for the values of the neutron star parameters M, a when the region of the allowed radial and vertical frequencies of the string loops cover all the observed twin HF QPO data. The situation is illustrated in Fig. 2 (left), where the example of the $\nu_r - \nu_\theta$ dependence is given for properly chosen values of the spacetime parameters M, a. The limiting lines correspond to the limiting values of $\omega \in \langle -1, 1\rangle$. The spin parameter has been taken at the value of $a = 0.5$ corresponding to the limit of applicability of the Hartle–Thorne theory (Urbanec et al., 2013).

Finally, the restrictions on the spacetime parameters M, a of the neutron stars in the atoll sources 4U 1636-53, predicted by the string loop oscillation model, are presented in Fig. 2 (right). The allowed region of the neutron star spacetime parameters is determined by a numeric procedure searching for the values of M, a parameters allowing to cover all the twin HF QPO data of the given source with the whole range of the string loop parameter ω. For completeness, we have considered whole the range of the neutron star spin, $a < 0.7$, as predicted by the fully general relativistic models of neutron stars (Lo and Lin, 2011). In the presented approach, restrictions on the string loop parameter ω are not discussed, as the whole range of $\omega \in (-1, +1)$ is allowed. In Figure 3, the radial profiles of the frequency of the radial and vertical harmonic oscillatory modes are presented in a typical situation enabling fitting of observational data, while limiting role of the neutron star surface is depicted.

We can see that the string loop oscillation model allows for the neutron star atoll source 4U 1636-53 its spin in the range $0.45 < a < 0.7$ and its mass in the range $2.5M_{\odot} < M < 2.9M_{\odot}$. These ranges seem to be in marginal agreement with the Hartle–Thorne theory of the neutron stars, and the limits implied on neutron star mass by realistic equations of state (Urbanec et al., 2013). In fact, the limiting value of $a = 0.45$ implies the mass of $M \sim 2.65\, M_{\odot}$ that can be explained by the very hard, mean-field equation of state.

5 CONCLUSIONS

We have demonstrated that the twin HF QPOs observed in the atoll 4U 1636-53 source can be explained by the string loop oscillation model introduced in (Stuchlík and Kološ, 2014b). This model, reflecting oscillations of string loops governed by interplay of the tension and angular momentum that can approximate magneto-plasma toroidal structures, has been for the atoll source applied in a way different to those related to simple systems of HF QPOs observed in microquasars or the peculiar neutron star XTE J1701-407 system, where resonant phenomena can be assumed. In the atoll source 4U 1636-53, the string loop oscillation model gives restrictions on the spacetime parameters M, a assuming no restrictions on the string loop parameter ω. We summarize that

- we cannot fit the observed data in the 4U 1636-53 source assuming only one string loop having a fixed value of the parameter ω, but we have to consider string loop with ω varied in the whole interval of allowed values $\omega \in (-1, +1)$.
- the neutron star has to be fast rotating, as the spin has to be in the range $0.45 < a < 0.7$,
- the neutron star has to be very massive, with mass parameter limited to the interval $2.5\,\mathrm{M}_{\odot} < M < 2.8\,\mathrm{M}_{\odot}$.

Since the neutron star has to be very massive, we can conclude that the application of the Kerr geometry in the fitting procedure is justified, as for the near-maximum-mass neutron stars the exterior Hartle–Thorne geometry has to be close to the exterior Kerr geometry, giving close predictions of the physical phenomena occurring in their vicinity. However, the predicted spin, $a \geq 0.45$, is too high for the Hartle–Thorne theory to be applicable. The applicability can be only marginal. Therefore, we could expect that some proper modifications of the spacetime parameters of the external field of the neutron star are possible due to an additional electromagnetic interaction of electrically charge string loops with the magnetic field of the neutron star (Tursunov et al., 2013, 2014).

Therefore, it is clearly worth to investigate the string loop oscillation model in more detailed way, concentrating on the conditions for creation of "magnetic" string loops due to the kinetic dynamo effect along the lines proposed in (Cremaschini and Stuchlík, 2013; Cremaschini et al., 2013).

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