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Some Aspects of Brany Kerr Spacetimes Relevant to Accretion Processes

Martin Blaschke[a], Zdeněk Stuchlík[b] and Petr Slaný[c]

Institute of Physics, Faculty of Philosophy & Science, Silesian University in Opava, Bezručovo nám. 13, CZ-746 01 Opava, Czech Republic
[a]Martin.Blaschke@fpf.slu.cz
[b]Zdenek.Stuchlik@fpf.slu.cz
[c]Petr.Slany@fpf.slu.cz

ABSTRACT
We consider equatorial motion of test particles around a rotating Kerr naked singularity in the Randall–Sundrum braneworld scenario and its implications for the properties of Keplerian accretion disks. We demonstrate existence of some unexpected phenomena related to properties of spacetimes having positive braneworld tidal charges. This new phenomenon can be an interesting explanation for extremely high energy cosmic radiation.

Keywords: Randall Sundrum – Brane-world

1 INTRODUCTION

In recent years, one of the promising approaches to the higher-dimensional gravity theories seems to be the string theory and particularly M-theory (Hořava and Witten, 1996; Hořava and Witten, 1996). This new idea is describing gravity as a truly higher-dimensional interaction becoming effectively 4D at low enough energies. Also these theories inspired so called braneworld models, in which the observable universe is a 3-brane on which the standard-model fields are confined, while gravity enters the extra spatial dimensions (Arkani-Hamed et al., 1998). The braneworld models provide an elegant solution to the hierarchy problem of the electroweak and quantum gravity scales, as these scales could become to be of the same order (TeV) due to large scale extra dimensions (Arkani-Hamed et al., 1998). Future collider experiments can test the braneworld models quite well, including even the hypothetical mini black hole production (Dimopoulos and Landsberg, 2001). The braneworld models could be tested observationally since they predict relevant astrophysically important properties of black holes. Gravity can be localized near the brane even with a non-compact, infinite size extra dimension with the warped spacetime satisfying the 5D Einstein equations as shown by Randall and Sundrum (1999). The rotating brany black hole spacetimes are represented by the Kerr-Newman geometry (without an electromagnetic field). The standard studies of black hole and naked-singularity geodetical motion (Stuchlík, 1981; Stuchlík and Calvani, 1991; Stuchlík and Hledík, 2000) can thus be fully applied for brane-world black holes and naked singularities with positive tidal charge.

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2 ORBITAL MOTION IN THE BRANEWORLD KERR SPACETIMES

Using standard Boyer–Lindquist coordinates (t, r, θ, φ) and geometric units $(c = G = 1)$, we can write the line element of a rotating (Kerr) black hole or naked singularity on the 3D-brane in the form

$$\mathrm{d}s^2 = -\left(1 - \frac{2Mr - b}{\Sigma}\right)\mathrm{d}t^2 - \frac{2a(2Mr - b)}{\Sigma}\sin^2\theta\,\mathrm{d}t\,\mathrm{d}\varphi + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\,\mathrm{d}\theta^2 + \left(r^2 + a^2 + \frac{2Mr - b}{\Sigma}a^2\sin^2\theta\right)\sin^2\theta\,\mathrm{d}\varphi^2\,, \quad (1)$$

where

$$\Delta = r^2 - 2Mr + a^2 + b\,, \quad (2)$$

$$\Sigma = r^2 + a^2\cos^2\theta\,, \quad (3)$$

M and $a = J/M$ are the mass parameter and the specific angular momentum of the background, and the braneworld parameter b, called the “tidal charge”, represents the imprint of non-local (tidal) gravitational effects of the bulk space (Aliev and Gümrükçüoğlu, 2005). The physical “ring” singularity of the braneworld rotating black holes (and naked singularities) is located at $r = 0$ and $\theta = \pi/2$, as in the Kerr spacetimes.

The form of the metric (1) is the same as that of the standard Kerr–Newman solution of the 4D Einstein–Maxwell equations, with the tidal charge b being replaced by the squared electric charge Q^2 (Misner et al., 1973). The following discussion can then be separated into these cases:

a) $b = 0$ in which we are dealing just with the standard Kerr metric.
b) $b > 0$ in which we are dealing with the standard Kerr–Newmann metric.
c) $b < 0$ where we are in the domain of new physics.

In the brany K-N spacetimes the geodetic motion is also relevant to charged test particles.

3 EFFECTIVE POTENTIAL AND RADIAL FUNCTION

The radial function $R(r)$ of the geodesic motion is defined by:

$$R(r) \equiv -\mathrm{sign}(m) + \frac{E^2 g_{\varphi\varphi} + 2ELg_{t\varphi} + L^2 g_{tt}}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}\,, \quad (4)$$

and the effective potential of the brany Kerr spacetimes takes the form:

$$V_{\mathrm{Eff}}(r, a, b, L) = \frac{-aL(b - 2r) \pm r\sqrt{\Delta}\sqrt{L^2r^2 + r^4 + a^2\left(r^2 + 2r - b\right)}}{r^4 + a^2\left(r^2 + 2r - b\right)}\,, \quad (5)$$

where L is the specific angular momentum as measured by an observer at infinity, E is the specific energy and m is the mass of the test particle. Circular motion is discussed in Stuchlík and Kotrlová (2009).

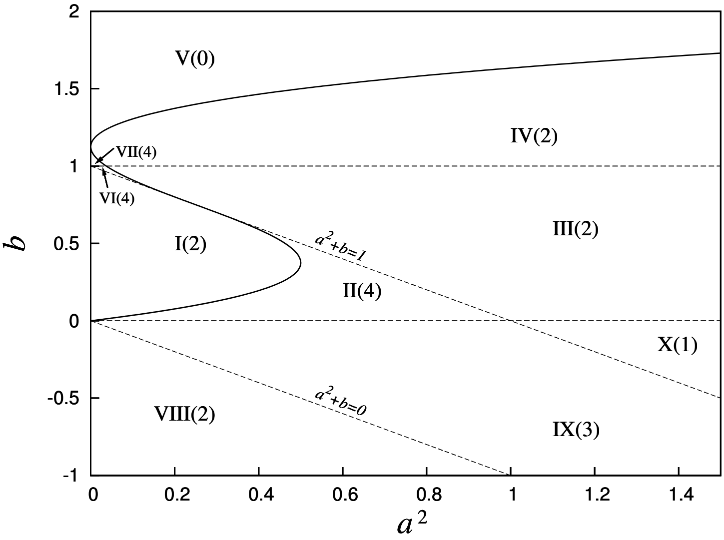

Figure 1. Brany Kerr black holes and naked singularities are divided into ten classes according to the properties of circular photon orbits. The corresponding regions of the b–a^2 plane are denoted by I–X; the number in parentheses gives the number of circular photon orbits in the respective class. See also Stuchlík (1981); Balek et al. (1989).

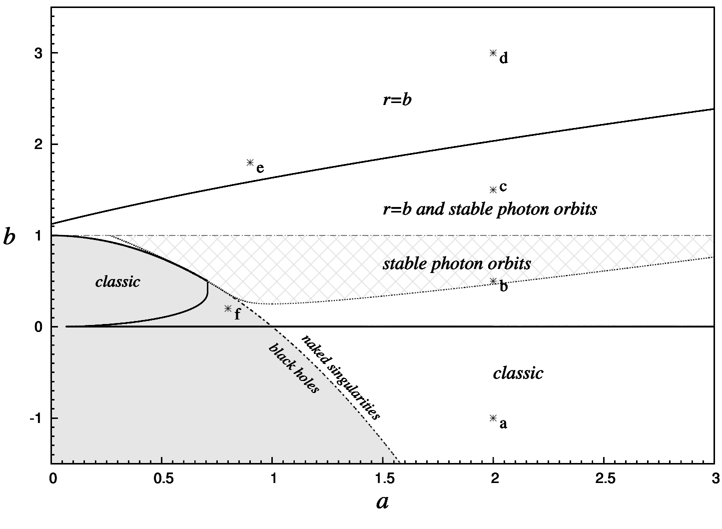

Figure 2. Classification of accretion disks with respect to parameters a and b. *Classic*: stands for those combinations of a and b where the ISCO coincides with the marginally stable orbit. *Stable photon orbit*: the ISCO for particles coincides with the stable circular photon orbit (the efficiency of accretion can then theoretically tend to infinity). *Stable photon orbit and* $r = b$: the ISCO is located at $r = b$ and the effective potential has a minimum for all positive values of L (this minimum is always higher than $r = b$ and is unimportant for accretion processes). *Region* $r = b$: the ISCO is located at $r = b$. The depicted star points correspond to chosen examples given in Fig. 3.

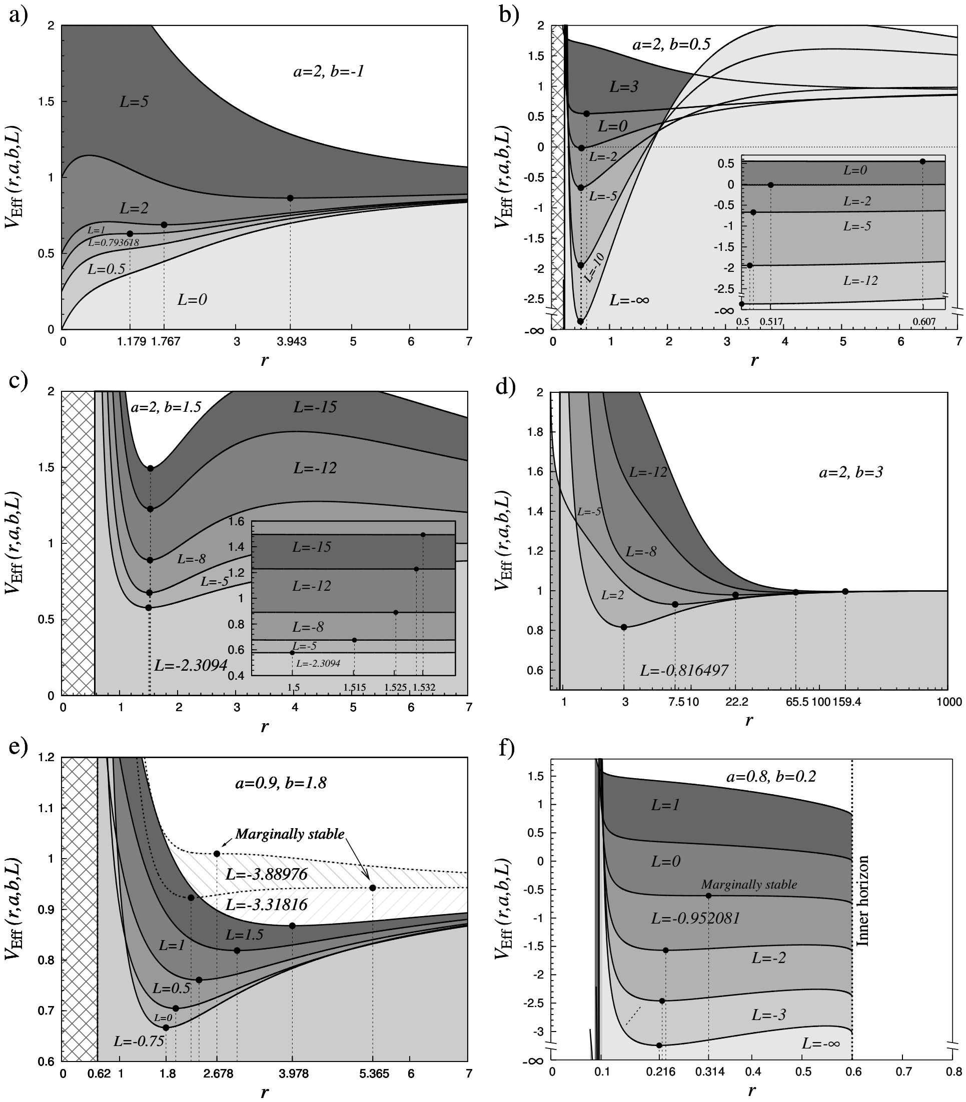

Figure 3. Examples of the effective potential from each region.

4 PHOTONS

In the case of photon orbits in the equatorial plane, the radial function $R(r)$ is determined by Eq. (4) with m set to zero (Schee and Stuchlík, 2009a,b):

$$\frac{R(r)}{E^2} = \frac{\left[r^2 - a(\lambda - a)\right]^2 - \Delta(\lambda - a)^2}{r^2 \Delta}, \tag{6}$$

where the impact parameter is defined by $\lambda = L/E$.

The photon orbits depend only on the impact parameter λ. The character of the photon motion is given by the number of circular orbits. We can distinguish ten cases of the brany Kerr spacetimes (Fig. 1).

5 EFFECTIVENESS OF ACCRETION

We discuss here some properties of thin Keplerian accretion disks. We will focus on disks orbiting naked singularities. The circular orbits can exist from infinity down to the radius of the limiting circular photon orbit, determined by the condition

$$r^2 - 3r + 2b \pm 2a\sqrt{r-b} = 0\,. \tag{7}$$

At this point E goes to $\pm\infty$ and L goes to $\pm\infty$, but the impact parameter $\lambda = L/E$ remains finite.

The loci of the stable circular orbits are given by the condition

$$\frac{\partial^2 R}{\partial r^2} \leq 0\,, \tag{8}$$

where the case of equality corresponds to the r coordinates of the marginally stable circular orbits $r_{\rm ms}$. This procedure of finding the marginally stable orbit as an inflexion point of the effective potential given by the condition (8), is what we will be calling a “standard treatment”. We obtain[1]

$$r\left(6r - r^2 - 9b + 3a^2\right) + 4b\left(b - a^2\right) \mp 8a\left(r-b\right)^{3/2} = 0\,. \tag{9}$$

This standard treatment works perfectly for the black holes, but as we shall demonstrate, does not work as well for counter-rotating disks around naked singularities.

The innermost stable circular orbit (ISCO) does not always correspond to the marginally stable orbit defined by Eq. (9).[2] This is demonstrated in Fig. (3, e) where we have depicted the effective potentials $V_{\rm Eff}(r, a, b, L)$. We can clearly see that sometimes the marginally stable orbit defined by Eq. (9) is not the innermost stable circular orbit. The ISCOs are actually located at $r = b$. The reason for this is that there can be a stable circular orbit at $r = b$, but not at $r < b$. This makes it possible to have an ISCO which is not an inflexion point of the radial function (4), which is the reason why the “standard treatment” (8) has to be treated very carefully. Of course, for accretion processes, the marginally stable circular orbits, i.e. the stable orbits with lowest energy, are relevant as the orbiting matter loses energy (and angular momentum) during accretion.

In the Figure 2 we have shown the classification of parameter space spanned by spin a and tidal charge b. This parameter space is divided into several areas according to following physical properties:

(1) existence of stable circular orbits in spacetime,

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[1] Formally the same results, relevant for Kerr–Newman spacetime, can be found in Aliev and Galtsov (1981).

[2] In some of the naked-singularity spacetimes (Reisner–Nordström, Kehagias–Sfetsos), two marginally stable orbits (ISCO and OSCO) can appear, (Pugliese et al., 2013; Stuchlík et al., 2014; Stuchlík and Schee, 2014; Vieira et al., 2014). However, this is not the case for the Kerr spacetimes (Stuchlík, 1980). See also Favata (2011).

(2) existence of ISCO at $r = b$ which is different than marginally stable orbit found by classic treatment,

(3) existence of ISCO which is identical with marginally stable orbit found by classic treatment.

Most interesting situation in Fig. 2 is lightly shaded area, where there are no present any ISCO's or marginally stable orbits. All orbits are stable up to a photon circular orbit, what is new phenomenon which can theoretically leads to unbound effectiveness of accretion.

6 CONCLUSIONS

We have shown an interesting new behaviour of the effective potential with regard to the stable circular photon orbits. These stable orbits can exist in the case of naked singularities in the Randall–Sundrum II brane-world scenario and in the case of classical Kerr–Newman naked singularities with quite a large amount of charge. This new phenomenon can be an interesting explanation for extremely high energy cosmic radiation.

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