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Compton scattering in strong gravity

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ABSTRACT
We present a new numerical code for radiation transport in strong gravity regime that includes arbitrary emission and absorption mechanisms and also electron scattering. We give a brief description of the methods employed. A simple example of a possible use is presented that also illustrates the effect of light bending on the comptonized thermal spectra.

Keywords: Radiation transport – relativity – Compton scattering

1 INTRODUCTION

There has been a growing interest in radiation transport codes in astronomy since the pioneering paper of Shakura and Sunyaev (1973) and there already exists plentiful of such codes that treat the problem of high photon and electron energies with different level of accuracy. X-ray sources usually contain a compact object as their central engine and so not only special relativistic but also general relativistic effects have to be properly included in the accurate and physically realistic treatment of radiation transport. Such codes must necessarily employ the emitter-to-observer scheme, which is naturally more computationally expensive. This seem to be the main reason why such codes have been developed only quite recently. The most advanced codes have been presented by Dolence et al. (2009) and by Schnittman and Krolik (2013).

In this paper, we introduce our own code for general relativistic radiation transport that includes arbitrary emission and absorption mechanisms and also electron scattering while properly taking into account all GR effects. We demonstrate the capabilities of the code on a simplified model of an accretion disk and comptonizing corona. Such an example will help us to quantify the effect of gravitation light bending on resulting spectra of comptonized thermal radiation and so to stress and justify the importance of GR treatment.

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2 PHOTON GEODESIC MOTION

Motion of photons in the Kerr spacetime can be solved for using several approaches. For instance, the geodesic equation can be numerically integrated to find a photon trajectory (e.g. Dovčiak et al., 2004), or one can proceed using Hamilton equations (e.g. Schnittman and Bertschinger, 2004) or one can write down the equation of motion and try to find a solution to it, which is possible in terms of elliptic integrals (e.g. Li et al., 2005).

Following Carter (1968), Bardeen et al. (1972) and Merloni et al. (1999), we can write down equations of motion for a photon in a separable form. In Boyer–Lindquist coordinates, where we employ a substitution of $m = \cos\theta$, they look like

$$\rho^2 \frac{\mathrm{d}r}{\mathrm{d}\lambda} = \pm\sqrt{R(r)}\,, \tag{1a}$$

$$\rho^2 \frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = \pm\sqrt{M(m)}\,, \tag{1b}$$

$$\rho^2 \frac{\mathrm{d}\varphi}{\mathrm{d}\lambda} = -a + \frac{l}{1-m^2} + \frac{a}{\Delta}\big(r^2 + a^2 - a\,l\big)\,, \tag{1c}$$

$$\rho^2 \frac{\mathrm{d}t}{\mathrm{d}\lambda} = -a^2\big(1-m^2\big) + a\,l + \frac{r^2+a^2}{\Delta}\big(r^2 + a^2 - a\,l\big)\,, \tag{1d}$$

where

$$R = \big(r^2 + a^2 - al\big)^2 - \Delta\big((l-a)^2 + Q\big)\,, \tag{2}$$

$$M = Q - \frac{l^2\,m^2}{1-m^2} + a^2 m^2\,, \tag{3}$$

$$\Delta = r^2 - 2r + a^2\,, \tag{4}$$

$$\rho^2 = r^2 + a^2\,m^2 \tag{5}$$

and $q = Q/E_\infty^2$ is scaled Carter's constant, $l = L_z/E_\infty$ is conserving angular momentum about the black hole z-axis.

With the help of above equation the photon trajectory can be directly calculated if we know the initial conditions, *i.e.* a point on the trajectory and the direction of the photon. As an example, we can think of a photon that has been emitted from the surface (photosphere) of an accretion disk. If photons are emitted isotropically, we can randomly choose the initial direction at a given place, calculate photon's 4-momentum and constants of motion and iterate numerically Eq. (1). We will describe the exact procedure in Section 5.

3 COMPTON SCATTERING

If photons have to propagate through a non-empty environment, they may encounter collisions with other particles. In astrophysics, the most relevant process of this type is a collision of a photon with a (quasi-)free particle, usually with an electron. Such a process is called Compton scattering after Arthur Holly Compton who observed it for the first time in 1923 while scattering X-ray photons on stationary electrons (Compton, 1923). This experiment played an important role in persuading physicists that light can behave as a stream of particle-like objects (quanta) whose energy is proportional to the frequency. Eventually, Compton earned Nobel Prize for his discovery five years later.

Often, the term Compton scattering is used to describe the original process in which an energetic electron scatters off an electron at rest. However, an opposite situation may happen as well: a lower energy photon is scattered to higher energy by a relativistic electron. This process is refereed to as Inverse Compton scattering and it is of great importance in astrophysics. However, as basically both processes share the same mechanism, we are using the term Compton scattering for both throughout this paper.

The energy of the photon after the scattering event is different from its initial energy, because some of it is exchanged with the electron. The ratio of photon energy after and before the collision is

$$P(E_\gamma, \theta) = \frac{1}{1 + (E_\gamma / m_\mathrm{e} c^2)(1 - \cos\theta)}\,, \tag{6}$$

where E_γ is the original photon energy before collision, m_e is mass of an electron and $\cos\theta$ is the scattering angle, which itself is given by Klein–Nishina differential cross-section formula. For incident photon energies much lower than the electron rest energy, $\epsilon \ll m_\mathrm{e}c^2$, the energy-dependent Klein–Nishina formula can be replaced by a simpler Thomson approximation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{2} r_0^2 \big(1 - \cos\theta\big)\,, \tag{7}$$

which does not depend on the photon energy. We should keep in mind that the photon momentum must be Lorentz-transformed to the electron rest frame, so if we have a distribution of thermal electrons, there is a factor γ that modifies the photon's energy in the frame of the electron. Still, if we consider initial photons with energy $\lesssim 1$ keV and $\lesssim 10^8$ K electron temperature, Thomson approximation can be used safely.

4 COMPGR

In this article, we are introducing a new code, `COMPGR`, that is capable of calculating effects of Compton scattering in the regime of strong gravity.

`COMPGR` combines two existing codes, a code for Compton scattering COSMOC (Adámek and Bursa, 2014) and a code for relativistic ray-tracing SIM5 (Bursa et al., 2004), in a readily usable package. `COMPGR` finds its use in situations where it is necessary to accurately compute comptonization effects on radiation in a close vicinity of a black hole, where strong gravitational light bending causes photons to follow curved geodesic trajectories.

The code generates photons according to the specified geometry and distribution and those are then propagated along geodesics based on their initial conditions. At each step of the trajectory, the total optical depth to scattering along the travelled path is increased by

$$\mathrm{d}\tau = \kappa_\mathrm{es}\,\rho\,\mathrm{d}l \tag{8}$$

and the probability of scattering is evaluated as

$$P_\mathrm{SC} = 1 - e^{-\tau}\,. \tag{9}$$

When the photon does scatter off an electron, we make a coordinate transformation from coordinate basis to fluid local rest frame (LRF). In LRF we determine the temperature of the fluid and corresponding electron energy distribution. This gives us particular electron velocity and direction and after transforming into the rest frame of the electron, we perform the scattering calculation. The electron velocity is taken from isotropic Maxwell–Jüttner distribution (Jüttner, 1911)

$$f_{\mathrm{MJ}}(\gamma) = \frac{\gamma^2 \beta}{\theta_T \, \mathrm{K}_2(1/\theta_T)} \exp(-\gamma/\theta_T)\,, \tag{10}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$, $\theta_T = kT/m_{\mathrm{e}}c^2$, and K_2 is modified Bessel function. After we determine new photon energy and direction, we make a back-transformation from electron rest frame to LRF and from LRF to the coordinate frame, we update photon momentum and constants of motion and the photon is followed along the new trajectory until it scatters again or until it either escapes sufficiently far or hits the black hole or is destroyed by some other means depending on the physical setup.

The code can also handle optically thin emission and absorption, such as bremsstrahlung or synchrotron. In the fluid rest frame, the radiation transport equation is

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}l} = j_\nu - \alpha_\nu \, I_\nu\,, \tag{11}$$

where $\mathrm{d}l$ is the path element and I_ν, j_ν and α_ν are respectively the specific intensity, emissivity and absorption of the fluid that are given by the specific radiation process. The emission and absorption processes are integrated along the photon path so that the intensity of a particular photon bundle is updated at each step.

5 EXAMPLE

We demonstrate the code on a simple example of a thin Shakura–Sunyaev-type accretion disk that is surrounded by a optically thin hot corona. We employ a very simple physical setup that shall demonstrate the effect of light bending on the scattered spectra.

In the case of an accretion disk surrounding a black hole the disk produces a thermal spectrum with typical energy of few kiloelectronvolts or less (depending on the mass of the black hole). The lower energy photons emitted from the disk surface are scattered to higher energies by relativistic electrons in the surrounding corona. This effect is believed to cause the power law component observed in X-ray spectra of accreting black holes.

In this numerical experiment, we use a non-rotating Schwarzschild black hole with an accretion disk around it. The disk has a standard inner edge at $6M$ and we will use a power-law temperature profile $T = T_0\, r^{-3/4}$ for it (flux $F(r) \sim r^{-3}$), where T_0 is fixed at 3 keV.

In addition, there is a radial and spherically symmetric wind with constant temperature $T_{\mathrm{e}} = 10^8$ K, constant $\dot{M}$ and constant radial velocity $v_{\mathrm{e}} = 0.1\,c$. In our simplified setup, the wind originates from the black hole, but more realistically we can imagine a wind that is fed by the material from the disk. Density of the wind decreases with radius as

$$\rho = \frac{\dot{M}}{4\pi\, r^2\, v_{\mathrm{e}}}\,. \tag{12}$$

The total optical depth of the wind along the line of sight is

$$\tau = \int_{r_0}^{\infty} \rho\, \sigma_{\mathrm{T}}\, \mathrm{d}r\,, \tag{13}$$

which combined with the previous expression gives the density

$$\rho = \frac{r_0}{r^2}\,\frac{\tau}{\sigma_{\mathrm{T}}} \tag{14}$$

simply as a function of radius and total optical depth. r_0 is the radius of the wind base, that is of the black hole horizon in our case, $r_0 = 2M$. Photons are scattered in the wind, but we assume that after 10 such events the photon is lost (it is likely to be absorbed by an ion).

To illustrate the effect of light bending, we perform two simulations for each setup: one uses full GR with geodetic photon trajectories, in the second one we follow the photon path along straight lines in the original direction of emission while preserving other GR effects. True photon geodesic motion has been described in Section 2. In the simulation, where we assume straight photon paths, we replace Eq. (1) by its limit version for $M = a = 0$ and we solve it. That way, we can follow the marked direction of emitted photon as it would go through a flat spherically symmetric space until the trajectory escapes far enough from the black hole, ends up in the black hole, ends up in the disk, or we encounter a scattering event in corona. No other changes are made and the scattering, g-factor evaluation, etc., still use standard Kerr metric and GR formalism, so we can say that the the eventual difference in results is solely due to photons taking bended or straight paths.

What can we expect? When the corona is not considered and photons travel from the disk to the observer freely without any scatterings, we expect the ‘flat’ spectrum to be identical to ‘GR’ spectrum at lower energies that are contributed by photons coming from cooler outer parts of the disk, where relativistic effects do not play much role. At higher energies, on the other hand, we expect the ‘flat’ spectrum to be slightly harder than the ‘GR’ spectrum simply because the photons come from the disk to the observer, who has a fixed inclination of 60°, on average at slightly more grazing angles than it is the case in GR, where light bending effect allows for escaping angles nearly parallel to the disk normal from parts of the disk. Larger emission angles mean larger Doppler boost and thus the spectrum should harden. The corresponding two spectra (‘GR’ and ‘flat’) are compared in Fig. 1, where the difference due to light bending can clearly be seen.

With corona present, we eventually expect the difference to go in the opposite direction – the ‘flat’ spectra should be softer as the corona becomes optically thicker. The density of the corona decreases with square of the distance from the black hole, so the largest scattering probability is very close to the black hole. If photons follow true GR geodesics they feel the ‘attractive force’ of the black hole’s gravity and the strongly bended trajectories cause photons to stay around the black hole longer then it is the case for ‘flat’ photons that do not feel such effect. Again, that acts mostly on the most on energetic photons coming from inner parts of the disk that have more chances to upscatter and harden the ‘GR’ spectra. Figure 2 shows how the spectrum evolves with increasing optical depth $\tau = 1$, $\tau = 5$ and $\tau = 10$. When the optical depth is too high and the corona is largely optically thick, the difference in the two spectra should diminish.

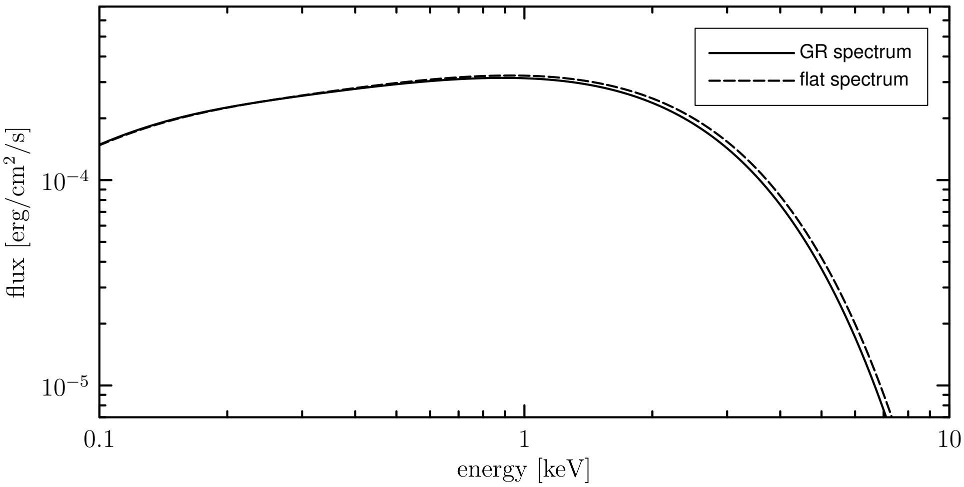

Figure 1. Observed spectra when no corona is assumed. Photons travel from the disk to the observer freely without any scatterings. Solid line represents the *true* spectrum with all GR effects taken account properly. Dashed line shows how would the spectrum look like if there was no relativistic light bending and photons were following straight lines as in a flat space (all other GR effects are preserved).

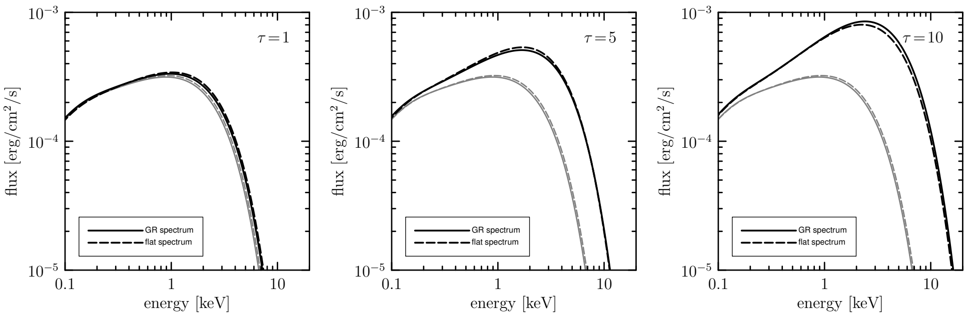

Figure 2. Observed spectra (*lines in bold black*) for total optical depths $\tau = 1$ (*left*), $\tau = 5$ (*middle*) and $\tau = 10$ (*right*). Gray lines show the spectra from Fig. 1 without corona for reference. Corona temperature is 10^8 K and its density decreases with square of radius.

6 SUMMARY

We have presented a new numerical code, `COMPGR` along with some technical details behind its treatment of general relativistic radiation transport. Its capabilities include namely Compton scattering with proper treatment of geodesic photon propagation between collision events and as such it finds its use in setups involving scattering regions close to a black hole, where gravitational effects on light propagation play a strong role.

We have demonstrated the importance of GR effect on the simple example of scattering soft thermal photons in the radial wind of hot electrons. The results clearly justify the need for a proper GR treatment of radiation transport.

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REFERENCES

Adámek, K. and Bursa, M. (2014), Simulating Compton Scattering using Monte Carlo method: COSMOC library, in Z. Stuchlík, G. Török and T. Pecháček, editors, *Proceedings of RAGtime 14–16: Workshops on black holes and neutron stars, Opava, Prague, 18–22 September/15–18 July/11-19 October '12/'13/'14*, pp. 1–10, Silesian University in Opava, Opava, ISBN 978-80-7510-126-6, in this proceedings.

Bardeen, J. M., Press, W. H. and Teukolsky, S. A. (1972), Rotating Black Holes: Locally Nonrotating Frames, Energy Extraction, and Scalar Synchrotron Radiation, *Astrophys. J.*, **178**, pp. 347–370.

Bursa, M., Abramowicz, M. A., Karas, V. and Kluźniak, W. (2004), The Upper Kilohertz Quasi-periodic Oscillation: A Gravitationally Lensed Vertical Oscillation, *Astrophys. J. Lett.*, **617**, pp. L45–L48, arXiv: astro-ph/0406586.

Carter, B. (1968), Global Structure of the Kerr Family of Gravitational Fields, *Physical Review*, **174**, pp. 1559–1571.

Compton, A. H. (1923), A Quantum Theory of the Scattering of X-rays by Light Elements, *Physical Review*, **21**, pp. 483–502.

Dolence, J. C., Gammie, C. F., Mościbrodzka, M. and Leung, P. K. (2009), grmonty: A Monte Carlo Code for Relativistic Radiative Transport, *Astrophys. J. Suppl.*, **184**, pp. 387–397, arXiv: 0909.0708.

Dovčiak, M., Karas, V. and Yaqoob, T. (2004), An Extended Scheme for Fitting X-Ray Data with Accretion Disk Spectra in the Strong Gravity Regime, *Astrophys. J. Suppl.*, **153**, pp. 205–221, arXiv: astro-ph/0403541.

Jüttner, F. (1911), Das Maxwellsche Gesetz der Geschwindigkeitsverteilung in der Relativtheorie, *Annalen der Physik*, **339**, pp. 856–882.

Li, L.-X., Zimmerman, E. R., Narayan, R. and McClintock, J. E. (2005), Multitemperature Blackbody Spectrum of a Thin Accretion Disk around a Kerr Black Hole: Model Computations and Comparison with Observations, *Astrophys. J. Suppl.*, **157**, pp. 335–370, arXiv: astro-ph/0411583.

Merloni, A., Vietri, M., Stella, L. and Bini, D. (1999), On gravitomagnetic precession around black holes, *Monthly Notices Roy. Astronom. Soc.*, **304**, pp. 155–159, arXiv: astro-ph/9811198.

Schnittman, J. D. and Bertschinger, E. (2004), The Harmonic Structure of High-Frequency Quasi-periodic Oscillations in Accreting Black Holes, *Astrophys. J.*, **606**, pp. 1098–1111, arXiv: astro-ph/0309458.

Schnittman, J. D. and Krolik, J. H. (2013), A Monte Carlo Code for Relativistic Radiation Transport around Kerr Black Holes, *Astrophys. J.*, **777**, 11, arXiv: 1302.3214.

Shakura, N. I. and Sunyaev, R. A. (1973), Black holes in binary systems. Observational appearance., *Astronomy and Astrophysics*, **24**, pp. 337–355.