Proceedings of *RAGtime 14–16, 18–22 Sept./15–18 July/11–19 Oct., '12/'13/'14, Opava, Prague, Czech Republic*
Z. Stuchlík, G. Török and T. Pecháček, editors, Silesian University in Opava, 2014, pp. 83–92

Evolutionary tracks of millisecond pulsars with low-mass companions

Filip Ficek[1], Mieszko Rutkowski[1] and Włodek Kluźniak[2]

[1]Institute of Physics, Jagiellonian University, ul. Łojasiewicza 11, 30-348 Kraków, Poland
[2]Nicolaus Copernicus Astronomical Center, Bartycka 18, 00-716 Warszawa, Poland

ABSTRACT
We consider the evolution of millisecond radio pulsars in binary systems with a main-sequence or evolved stellar companion. Evolution of non-accreting binary systems with "eclipsing" millisecond pulsars was described by Kluźniak, Czerny and Ray (1992) who predicted that systems like the one containing the Terzan 5 PSR 1744-24A will in the future become accreting low mass X-ray binaries (LMXBs), while PSR 1957+20 may evaporate its companion. The model presented in the current paper gives similar results for these two objects and allows to obtain diverse evolutionary tracks of millisecond pulsars with low mass companions (black widows). Our results suggest that the properties of many black widow systems can be explained by an ablation phase lasting a few hundred million years. Some of these sources may regain Roche lobe contact in a comparable time, and become LMXBs.

Keywords: millisecond pulsar – redback – black widow – binary evolution – ablation – LMXBs – gravitational waves

1 INTRODUCTION

Millisecond pulsars are probably intimately connected with LMXBs, as was realized soon after their discovery: it was suggested that millisecond pulsars have been spun up in LMXBs and will end their history in the radio pulsar phase (Radhakrishnan and Srinivasan, 1982; Alpar et al., 1982). However, with the discovery of the eclipsing pulsars it was realized that some millisecond pulsars currently ablating their companions may re-enter the LMXB phase in a later epoch (Bisnovatyi-Kogan, 1989; Ergma and Fedorova, 1991; Kluźniak et al., 1992). Recent discoveries of many ablating binary systems have led to a rekindling of these ideas, and to the necessity of explaining the evolutionary status of these black widows and redbacks, as they are called (e.g. Roberts et al., 2014).

We are presenting an evolutionary model describing a binary system composed of a pulsar and its stellar companion. The model includes effects like gravitational wave emission by the binary, ablation of the companion, and pulsar spindown. In general, part of the ablated matter may accrete onto the neutron star and another part may leave the system. The computed evolutionary tracks begin with the pulsar turn-on at the conclusion of the standard epoch of accretion in a semi-detached phase. Throughout most of the computed

978-80-7510-127-3

evolutionary history, the separation between the pulsar and the companion star is large enough for the latter to be below its Roche lobe. Therefore the only mechanism of mass loss considered in our model is ablation by the pulsar wind.

2 MODEL DESCRIPTION

The period of a binary system including a pulsar of mass M and its companion of mass m is

$$P = \frac{2\pi J^3(M+m)}{G^2 M^3 m^3} , \tag{1}$$

where J denotes total orbital angular momentum. The rate of change of the companion mass m is assumed to be proportional to the spin-down flux

$$\dot{m} \propto \frac{\dot{E}}{4\pi d^2} m^a P^b , \tag{2}$$

where $\dot{E}$ is the energy loss of the pulsar primary owing to its spindown, d is the separation between the primary and the secondary, and a, b are model dependent exponents. In the simple model assumed in Kluźniak et al. (1992) $a = b = 0$. However, in Brookshaw and Tavani (1995) one may find $a = 1/6$ and $b = -4/3$. We will adopt the latter values. The change of mass of the primary is in principle connected with $\dot{m}$ as $\dot{M} = -\beta\dot{m}$. The coefficient β describes how much of the mass lost by the companion is accreted by the neutron star, and how much is lost from the binary in a wind, thus $0 \leq \beta \leq 1$ with 0 corresponding to no accretion and 1 to no wind. We will take $\beta = 0$.

The change of angular momentum [first term in Eq. (5)] is connected with two processes: emission of the gravitational waves (GW) and mass loss from system. We take the rate of angular momentum loss to gravitational waves to be described by (e.g. Shapiro and Teukolsky, 1983)

$$\dot{J}_{\rm GW} = -\frac{256\pi^3}{5}\frac{G}{c^5}\frac{J^2}{P^3} . \tag{3}$$

If we assume that specific angular momentum carried away by a wind escaping from the system is $j = \alpha M J/[m(m+M)]$, we have

$$\dot{J}_{\dot{m}} = \alpha(1-\beta)\frac{MJ}{(M+m)}\frac{\dot{m}}{m} . \tag{4}$$

Both Equations (3) and (4) conribute to the rate of change of the angular momentum: $\dot{J} = \dot{J}_{\dot{m}} + \dot{J}_{\rm GW}$. By differentiating Eq. (1) with respect to time we get the rate of change of the period

$$\frac{\dot{P}}{P} = 3\frac{\dot{J}}{J} - \frac{2M + 3m}{M + m}\frac{\dot{M}}{M} - \frac{3M + 2m}{M + m}\frac{\dot{m}}{m} , \tag{5}$$

$$\dot{m} = \gamma \frac{\dot{E} m^{1/6} P^{-4/3}}{4\pi d^2} , \tag{6}$$

$$\dot{M} = -\beta \dot{m} , \tag{7}$$

$$\frac{\dot{J}}{J} = \frac{\alpha(1-\beta)M}{m + M}\frac{\dot{m}}{m} - \frac{256\pi^3 G J}{5c^5 P^3} . \tag{8}$$

Equations (5), (6), (7) and (8) constitute a system of first-order ordinary differential equations, which we proceed to solve with various assumptions and different initial conditions.

In the simple case of no accretion onto the primary star, negligible companion mass, $m \ll M$, and hence negligible gravitational wave emission, the equations reduce to (Kluźniak et al., 1992)

$$\frac{\dot{P}}{P} = 3(\alpha - 1)\frac{\dot{m}}{m} , \tag{9}$$

and can be easily integrated, yielding

$$P(m) \propto m^{3(\alpha-1)} . \tag{10}$$

With suitable initial conditions the evolutionary paths on the P vs. m plot described by this equation can be made to pass through the current positions of some of the known pulsars, e.g. PSR 1957-20 (see the Appendix, Fig. A1).

The source of the energy driving the ablation process is pulsar spindown. From the magnetic dipole formula (e.g. Shapiro and Teukolsky, 1983) we have

$$\dot{E} = -\frac{B^2 R^6 \Omega^4 \sin^2\theta}{6c^3} , \tag{11}$$

where B denotes the surface magnetic field near the pole, Ω is the pulsar spin rate (the pulsar period being $P_0 = 2\pi/\Omega$), R is the pulsar radius and θ denotes the angle between the magnetic and the rotation axes (for simplicity we take $\sin^2\theta = 1$ and $R = 10\,\mathrm{km}$). On the other hand we have $\dot{E} = I\Omega\dot{\Omega}$, where I is the moment of inertia of the pulsar. These two equations provide

$$\Omega(t) = \frac{\Omega_0}{\sqrt{2t/\tau + 1}} , \tag{12}$$

where $\Omega_0 = \Omega(0)$ is the initial angular velocity of the pulsar and $\tau = -\Omega(0)/\dot{\Omega}(0)$ is the characteristic age of the pulsar (at time t=0). Equations (11) and (12), are used in Eq. (6) to find $\dot{m}$ as a function of time.

When the secondary star is sufficiently close to the pulsar that it fills the Roche lobe, accretion through the inner Lagrangian point starts. This situation is not described by our model, although our tracks may bring the system to this point. When the radius of the companion is equal to Roche lobe radius, the relation between orbital period P and companion mass m is

$$P = 2\pi\sqrt{\frac{A^3}{B^3 G}m^{(3n-1)/2}}\,, \tag{13}$$

where $B \approx 0.462$. The values of n and A correspond to the radius of the companion through $r = Am^n$. For degenerate stars like white dwarfs, $n = -1/3$, and for a hydrogen white dwarf $A = 2.82 \times 10^4 M_\odot^{1/3}$ km (Shapiro and Teukolsky, 1983), while from Hamada and Salpeter (1961) one obtains $A = 8.80 \times 10^3 M_\odot^{1/3}$ km for a helium white dwarf. One may also obtain this coefficient for a carbon white dwarf, which is $A = 8.72 \times 10^3 M_\odot^{1/3}$ km (Hamada and Salpeter, 1961), it is almost indistinguishable from the helium one. Lines corresponding to Eq. (13) indicate where the evolutionary track may terminate in a Roche-lobe overflowing LMXB, depending on the companion type (Fig. 1).

3 RESULTS OF NUMERICAL CALCULATIONS

Using Mathematica, we solved numerically the system of four differential equations, i.e. Eqs. (5), (6), (7) and (8) discussed above. We consider a model with no accretion ($\beta = 0$), we assume $\gamma = 2.5 \times 10^4\ \mathrm{s}^{10/3}\ \mathrm{g}^{-1/6}$ (cf., Chen et al., 2013) and, following Kluźniak et al. (1992), we take $\alpha = 0.86$. For the initial point on the (m, P) plane we use one of two points on the track of Tauris and Savonije (1999), which describes the evolution of a LMXB with an evolved companion. For the PSR 1957+20 and B1744-74A (Terzan 5) tracks we use the starting point of Kluźniak et al. (1992), corresponding to the point at which magnetic braking is supposed to lose importance in the evolution of binaries with a main sequence companion. Current system parameters are taken from ATNF Database (2014); Manchester et al. (2005), and they can be found in Table 1, together with other data, for the six tracks which are presented in Fig. 1.

Derived times of evolution are $t_{\rm ev} \simeq 7 \times 10^8$ y for PSR 1957+20 and $t_{\rm ev} \simeq 5.5 \times 10^9$ y for Terzan 5. For PSR 1957+20, evolution is steady, whereas for Terzan 5 one can distinguish three stages of evolution. The first stage, when the evolution curve is nearly a straight line, lasts about 4.5×10^8 y. The second one, when the evolution path "turns downwards" on the P vs m plot, lasts 2.4×10^9 y. The last stage, when gravitational radiation is dominant, lasts 2.6×10^9 y. Objects with convex evolution curves evolve comparably fast: e.g. for J1807-2459A the evolution time is $t_{\rm ev} \simeq 5.7 \times 10^8$ y. The values of $t_{\rm ev}$ in parentheses in Table 1 (for the Terzan 5 pulsar and J1023+0038) correspond to the time it will take for the system to regain the line of Roche-lobe contact starting from the present position.

4 EVOLUTIONARY TRACKS

Evolution of the system depends on the ratio between angular momentum losses caused by ablation and gravitational wave emission. There seem to be three types of tracks.

In the case where gravitational waves emission can be neglected (like in the PSR 1957-20 system) the track is well described by the formula of Eq. (10). The system very nearly

Table 1. System parameters

Quantity	**PSR 1957+20**		**B1744-74A/Terzan 5**	
	Initial	Present	Initial	Present
P[hr]	2.9	9.2	3.0	1.82
$m[M_\odot]$	0.235	0.022	0.235	0.087
$M[M_\odot]$	1.7	1.7	1.4	1.4
$\dot{m}$[g/s]	-2.0×10^{17}	-7.6×10^{14}	-1.1×10^{17}	-3.1×10^{14}
P_0[ms]	0.92	1.60	1.95	11.56
B[G]	4.0×10^{8}		$1.3 \times 10^{9.0}$	
$\mu\left[\mathrm{G}\times \mathrm{cm}^3\right]$	4.0×10^{26}		1.3×10^{27}	
t_{ev}[y]	6.72×10^{8}		5.52×10^{9} $\left(5.90\times10^{9}\right)$	
Quantity	**J1807-2459A**		**J2241-5236**	
	Initial	Present	Initial	Present
P[hr]	1.75	1.71	1.75	3.50
$m[M_\odot]$	0.115	0.009	0.115	0.012
$M[M_\odot]$	1.4	1.4	1.4	1.4
$\dot{m}$[g/s]	-4.5×10^{15}	-2.6×10^{15}	-1.4×10^{16}	-1.1×10^{15}
P_0[ms]	2.91	3.06	1.98	2.19
B[G]	2.9×10^{8}		2.4×10^{8}	
$\mu\left[\mathrm{G}\times\mathrm{cm}^3\right]$	2.9×10^{26}		2.4×10^{26}	
t_{ev}[y]	5.60×10^{8}		7.88×10^{8}	
Quantity	**J1311-3430**		**J1023+0038**	
	Initial	Present	Present	Predicted
P[hr]	1.75	1.56	4.73	0.31
$m[M_\odot]$	0.115	0.008	0.136	0.061
$M[M_\odot]$	1.4	1.4	1.4	1.4
$\dot{m}$[g/s]	-3.6×10^{15}	-3.0×10^{15}	-2.2×10^{16}	LMXB
P_0[ms]	2.48	3.56	1.67	11.09
B[G]	2.0×10^{8}		7.9×10^{8}	
$\mu\left[\mathrm{G}\times \mathrm{cm}^3\right]$	2.0×10^{26}		7.9×10^{26}	
t_{ev}[y]	5.44×10^{8}		$\left(1.8\times10^{10}\right)$	

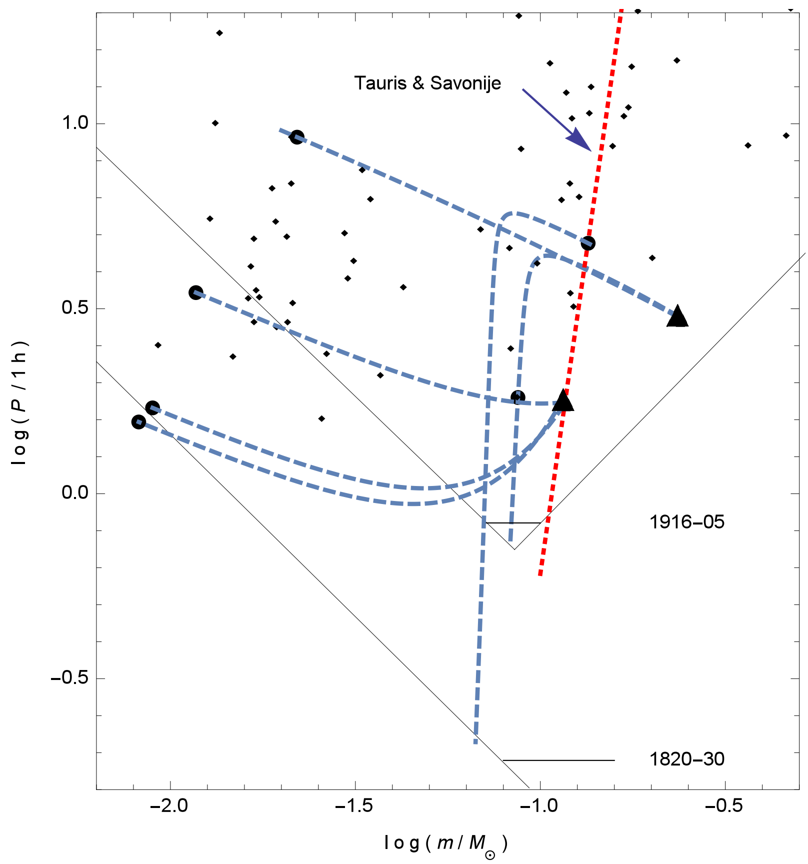

Figure 1. *Large dots* correspond to the present parameters of the observed pulsar systems for which the evolutionary tracks have been computed (*dashed blue lines*). *Small dots* are other objects taken from the ATNF Database (2014). *Solid lines* correspond to Roche lobe contact for a cold companion. The *short-dashed red line*, taken from Tauris and Savonije (1999), corresponds to LMXB evolution of a system with an evolved companion. Also shown (thin horizontal line segments) are the positions of two short-period LMXBs. The *filled triangles* mark plausible initial points of the evolutionary tracks.

follows a straight line on a $\log P$ versus $\log m$ plot. The slope of this line depends only on the parameter α. The track may be deflected a little bit due to vestigial gravitational wave emission.

Another possible track passes through the Terzan 5 pulsar B1744-24A. In the initial phase of system evolution the track is similar to the one described in the previous paragraph. The difference is that at a certain moment, owing to pulsar spindown, gravitational wave emission starts to dominate over ablation. If, from that point on, mass loss were neglected (i.e. the evolution were driven by GW emission alone), the track would be a vertical line on the $\log P$–$\log m$ plot. In fact, a residual effect of ablation is still felt, and the track deviates slightly in the direction of lower companion mass (to the left in the figures).

Neglecting mass loss from and mass transfer in the system ($\gamma = 0$ in Eq. (6)) one easily obtains the time elapsed in the evolution from binary period P_i to period P:

$$T = \frac{5c^5}{2048\pi^3 G J_i}\left(P^{8/3}P_i^{1/3} - P_i^3\right), \tag{14}$$

where J_i is the initial angular momentum (corresponding to P_i). Time scales of evolution obtained from this equation are similar to the numerical values for the nearly vertical tracks in Fig. 1.

Tracks similar to those described above were already obtained by Kluźniak et al. (1992). They cover situations in the limit where one of the effects, ablation or GW emission, dominates over the other along each major segment of the trajectory (although, as remarked above in Section 3, PSR B1744-74A spends most of its evolutionary time in transition between two such states). It seems that systems with an evolved very low mass companion ($m < 0.04 M_\odot$) cannot evolve this way. For instance, obtaining a "Terzan-like" evolution track for these systems leads to evolution time amounting to several dozens of billion years. A third type of evolutionary track seems to be required.

We have found evolutionary tracks connecting the currently observed binary parameters of the pulsars J2241-5236, J1807-2459A and J1311-3430 with a plausible initial point and having reasonable time scales of evolution. These evolutionary tracks are characterized by angular momentum loss to both GW emission and ablation effects, and have a convex shape on a $\log P$ versus $\log m$ plot (Fig. 1). Eventually, the separation of the system components becomes large enough that GW emission loses importance, and the track becomes parallel to that of PSR 1957+20.

5 DISCUSSION

We have considered the evolution of millisecond radio pulsars with binary low-mass companions assuming simple formulae for the ablation rate of the companion by the pulsar wind. For the starting point of each evolutionary track that we considered we have taken a plausible moment of pulsar turn-on in an erstwhile LMXB, either along the standard evolutionary curve familiar from discussion of cataclysmic variables and the period gap, i.e. a binary with a main-sequence companion (Paczyński and Sienkiewicz, 1983), or along an evolutionary track with an evolved companion Tauris and Savonije (1999). Pulsar turn-on (or turn-off) in (potentially) accreting low mass binaries was discussed in Kluźniak et al. (1988).

We have reproduced the results of Kluźniak et al. (1992) who performed a similar study for the only two known eclipsing pulsars at the time (PSR 1957+20 and B1744-74A in Ter 5), and found that there are periods of their evolutionary history in the ablation phase when either one or the other of two major angular momentum loss mechanisms dominates (mass loss from the system or GW emission). We note that evolutionary tracks that we now find based on the Brookshaw and Tavani (1995) evaporation formula, Eq. (2), imply shorter initial pulsar periods than previously obtained, this can be seen from a comparison of the entries in Table 1 with the description of tracks (b) and (e) in the Appendix, Fig. A1.

We find that we are able to reproduce the current positions of typical millisecond radio pulsars with a low mass binary companion, typically this involves an ablation phase lasting several hundred years. However, we find that for the majority of the black widow pulsars known today the relative importance of the two considered angular momentum loss mechanisms is comparable in their evolutionary history, i.e. unlike in the case of PSR 1957+20 and B1744-74A, neither GW emission nor mass loss dominates the other over major portions of the evolutionary track in the period-mass diagram (Fig. 1).

We confirm the conclusion of Kluźniak et al. (1992), who predicted that some ms pulsars may become accreting LMXBs at the end of their evolution. Two of the tracks presented in this paper end very close to the line of Roche-lobe contact, in the current position of PSR J1807-2459A and PSR J1311-3430. These two pulsars seem to be close to the end of a 5×10^8 y ablation phase.

We note that detailed binary evolutionary calculations, which included an ablation phase similar to the model considered here were presented recently in Chen et al. (2013).

ACKNOWLEDGEMENTS

We thank Dr. Thomas Tauris, as well as the anonymous referee, for many detailed comments on the manuscript. This work was supported in part by NCN grant 2013/08/A/ST9/00795.

REFERENCES

Alpar, M. A., Cheng, A. F., Ruderman, M. A. and Shaham, J. (1982), A new class of radio pulsars, *Nature*, **300**, pp. 728–730.

ATNF Database (2014), http://www.atnf.csiro.au/research/pulsar/psrcat/.

Bisnovatyi-Kogan, A. S. (1989), Two Generations of Low-Mass X-Binaries and Recycled Radio Pulsars, *Astrofizika*, **31**, p. 567.

Brookshaw, L. and Tavani, M. (1995), Outflow Hydrodynamics of Eclipsing Pulsar Binaries, *ASP Conference Series*, **72**, pp. 244–252.

Chen, H.-L., Chen, X., Tauris, T. M. and Han, Z. (2013), Formation of Black Widows and Redbacks–Two Distinct Populations of Eclipsing Binary Millisecond Pulsars, *Astrophys. J.*, **775**, 27, arXiv: `1308.4107`.

Ergma, E. V. and Fedorova, A. V. (1991), An 11-ms pulsar in the globular cluster TER 5 - A possible test for determination of the progenitor of millisecond pulsars, *Pisma v Astronomicheskii Zhurnal*, **17**, pp. 433–439.

Hamada, T. and Salpeter, E. (1961), Models for Zero-Temperature Stars, *Astrophys. J.*, **134**, pp. 683–698.

Kluźniak, W., Czerny, M. and Ray, A. (1992), From Milisecond Pulsars to X-Ray Binaries, *NATO Advanced Research Workshop on X-Ray Binaries and the Formation of Binary and Millisecond Radio Pulsars*, pp. 425–436.

Kluźniak, W., Ruderman, M., Shaham, J. and Tavani, M. (1988), Nature and evolution of the eclipsing millisecond binary pulsar PSR1957 + 20, *Nature*, **334**, pp. 225–227.

Manchester, R. N., Hobbs, G. B., Teoh, A. and Hobbs, M. (2005), The Australia Telescope National Facility Pulsar Catalogue, *Astronom. J.*, **129**, pp. 1993–2006, arXiv: `astro-ph/0412641`.

Paczyński, B. and Sienkiewicz, R. (1983), The minimum period and the gap in periods of cataclysmic binaries, *Astrophys. J.*, **268**, pp. 825–831.

Radhakrishnan, V. and Srinivasan, G. (1982), On the origin of the recently discovered ultra-rapid pulsar, *Current Science*, **51**, pp. 1096–1099.

Roberts, M. S. E., Mclaughlin, M. A., Gentile, P., Aliu, E., Hessels, J. W. T., Ransom, S. M. and Ray, P. S. (2014), Intrabinary shock emission from "black widows" and "redbacks", *Astronom. Nachr.*, **335**, pp. 313–317, arXiv: 1402.5507.

Shapiro, S. and Teukolsky, S. (1983), *Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects*, Wiley, Weinheim.

Tauris, T. and Savonije, G. (1999), Formation of millisecond pulsars, *Astronomy and Astrophysics*, **350**, pp. 928–944.

APPENDIX A: APPENDIX

For ease of reference, we reproduce Figure 1 and its caption from the pre-arXiv contribution of Kluźniak, Czerny and Ray (1992).

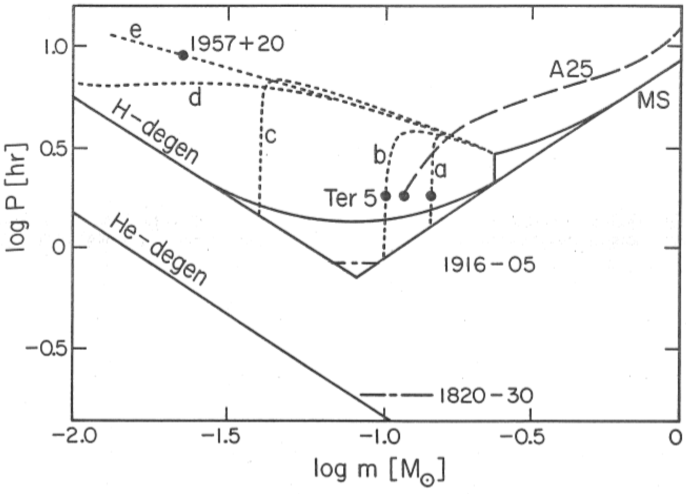

Figure A1. 'Figure 1. Possible evolutionary tracks of systems with "evaporative" mass loss. The decimal logarithm of the orbital period in hours is plotted versus the decimal logarithm of the mass of the companion in units of Solar mass. Likely location of the eclipsing pulsars (filed circles) as well as possible positions of the X-ray binaries 4U 1916-05 and 4U 1820-30 are also indicated (dash-dot-dash lines). The thick straight line segments correspond to systems with a main-sequence or a cold degenerate dwarf companion in Roche-lobe contact. According to the standard theory of their evolution, cataclysmic variables follow the thin curve (in the direction of decreasing companion mass, m). When this theory is applied to canonical LMXBs, the dotted tracks ensue, see Section 5 for details. The lines (a) through (e) differ only in the properties of the pulsar ablating its companion: in the strength of the magnetic dipole moment and in the initial value, P_0, of the rotational period of the neutron star. The values of P_0 and $\log(B/\text{Gauss})$, where $B \equiv \mu \times 10^{-18}\,\text{cm}^{-3}$, are respectively (a) 5.0 ms, 9.5; (b) 3.4 ms, 8.9; (c) 2.0 ms, 9.0; (d) 2.0 ms, 8.6; (e) 1.25 ms, 8.1. We assumed that 10 % of the energy flux impinging on the companion is converted into kinetic energy of the evaporative plume, and we took $\beta = 0.86$.'

N.B. The parameter "β" in the quoted caption corresponds to our α.