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Triggered oscillations and destruction of magnetized relativistic tori in 2D

Jaroslav Hamerský[1,2,a] and Vladimír Karas[1]

[1] Astronomical Institute, Academy of Sciences of the Czech Republic,
Boční II 1401, CZ-141 00 Prague, Czech Republic
[2] Charles University in Prague, Faculty of Mathematics and Physics,
V Holešovičkách 2, CZ-18000 Prague, Czech Republic
[a] jaroslav.hamersky@asu.cas.cz

ABSTRACT
Runaway instability operates under certain conditions in fluid tori around black holes. When active, it affects systems close to the critical (cusp overflowing) configuration. Here we start from our previous discussion of the role of runaway instability within a framework of an axially symmetric model of perfect fluid endowed with a purely toroidal magnetic field. The gradual accretion of material over the cusp transfers the mass and angular momentum onto the black hole, thereby changing the intrinsic parameters of the Kerr metric. By contributing to the total pressure, the magnetic field causes small departures from the corresponding non-magnetic configuration in the early phases of accretion. We showed that the toroidal magnetic component inside an accretion torus does not change the frequency of its oscillations significantly. We identified these oscillations as the radial epicyclic mode in our example. Nevertheless, these weak effects can trigger the runaway instability even in situations when the purely hydrodynamical regime of the torus is stable. On the other hand, in most cases the stable configuration remains unaffected, and the initial deviations gradually decay after several orbital periods. We showed examples of the torus evolution depending on the initial magnetization β, the slope q, and the spin a.

Perturbations in the vertical direction may lead to vertical oscillations. Here we propose that these oscillations could be enhanced (especially for an intermediate-mass black hole) by an orbiting star with a trajectory crossing the torus. First the oscillation of the torus material is triggered. Then the mass of the torus is dragged high enough above the equatorial plane and gradually accelerated along spin axis.

Keywords: Accretion: accretion discs – black-hole physics

1 INTRODUCTION

Toroidal equilibria of perfect fluid in permanent rotation were introduced a long time ago as an initial step on the way towards an astrophysically realistic description of accretion of gaseous material onto a black hole in active galactic nuclei and black hole binaries (Fishbone and Moncrief, 1976; Abramowicz et al., 1978; Pugliese et al., 2013). These axially symmetric and stationary solutions are subject to various types of instability (e.g. Abramowicz and Fragile, 2013). Here we concentrate on a global type of instability caused by an overflow of material over the cusp of a critical equipotential surface (Daigne and Mochkovitch, 1997; Abramowicz et al., 1998; Korobkin et al., 2013). It was suggested that this may lead to specific features that should be observable in the radiation emitted from such a system (Zanotti et al., 2003).

The effect of the mentioned instability can be catastrophic under certain conditions. In particular, a black-hole torus becomes runaway unstable if the angular momentum profile within the torus does not rise sufficiently fast with radius (Abramowicz et al., 1998; Lu et al., 2000). The role of general relativity effects on the runaway mechanism was studied in Font and Daigne (2002) in the context of gamma-ray burst sources. These authors found that by allowing the mass of the black hole to grow by accretion, the disc becomes unstable. However, the parameter space of the problem is much richer than what could be taken into account in early works. For example, the self-gravity of the fluid tends to act against the stability of non-accreting tori (Goodman and Narayan, 1988; Masuda et al., 1998; Montero et al., 2010; Korobkin et al., 2011). Furthermore, the spin parameter can play a role for accretion onto a rotating black hole. In astrophysically realistic models, an interplay of mutually competing effects have to be taken into account.

The role of magnetic fields is known to be essential for accretion. Even the Rayleigh-stable tori (Seguin, 1975) with a radially increasing profile, $\mathrm{d}l/\mathrm{d}R > 0$, become dynamically unstable because of turbulence in the presence of a weak magnetic field (Balbus and Hawley, 1991). Komissarov (2006) has developed a suitable analytical (toy) model of such a magnetized torus described by a polytropic equation of state in Kerr metric. In this model the magnetic field only enters the equilibrium solution for the torus as an additional pressure-like term. We employed this solution as an initial configuration, which we then perturbed and evolved numerically by using a two-dimensional numerical scheme (HARM; see Gammie et al., 2003).

From the mass estimates based on scaling relations that use high-frequency characteristic timescales, the mass of the black hole in M82 X-1, the bright X-ray source in the starburst galaxy M82, was estimated to be 400 solar masses (Pasham et al., 2014). Accretion tori of such intermediate-mass black holes could be perturbed by an orbiting star which could tear out considerable amount of mass above the equatorial plane and amplify the amplitude of vertical oscillations so that the overflow from an equipotential surface could occurs. As the mass approaches the horizon, centrifugal forces decelerate the mass, which causes an increase of pressure, and consequently this gas pressure accelerates the mass to create an outflow. We simulated this process effectively in 2D approach, assuming that the size of the perturbing star is approximately one fourth of the radial extent of accretion torus.

2 OSCILLATIONS OF MAGNETIZED RELATIVISTIC TORI

2.1 Axisymmetric accretion of magnetized fluid tori

The magnetized ideal fluid can be described by the energy-momentum tensor (e.g. Anile, 1989)

$$T^{\mu\nu} = \left(w + b^2\right) u^\mu\, u^\nu + \left(P_{\mathrm{g}} + \frac{1}{2} b^2\right) g^{\mu\nu} - b^\mu\, b^\nu\,, \tag{1}$$

where w is the specific enthalpy, P_g is the gas pressure, and b^μ is the projection of the magnetic field vector ($b^2 = b^\mu b_\mu$). From the energy-momentum tensor conservation, $T^{\mu\nu}_{\;;\nu} = 0$, it follows for a purely axially rotating fluid (Abramowicz et al. 1978; 2013)

$$\ln|u_t| - \ln|u_{t_\mathrm{in}}| + \int_0^{P_\mathrm{g}} \frac{\mathrm{d}P}{w} - \int_0^{l} \frac{\Omega\,\mathrm{d}l}{1-\Omega l} + \int_0^{\tilde{P}_\mathrm{m}} \frac{\mathrm{d}\tilde{P}}{\tilde{w}} = 0\,, \tag{2}$$

where u_t is the covariant component of the four-velocity (subscript "in" corresponds to the inner edge of the torus), $\Omega = u^\varphi/u^t$ is the angular velocity and $l = -u_\varphi/u_t$ is the angular momentum density. By assuming a suitable polytropic equation of state and the rotation law of the fluid, Eq. (2) can be integrated to obtain the structure of equipotential surfaces of the equilibrium configuration (Hamerský and Karas, 2013).

We assume that the above-described initial stationary state is pushed out of equilibrium. This leads to the capture of a small amount of material by the black hole, which increases the black-hole mass, and so the accretion occurs. Abramowicz et al. (1998) argued that tori with radially increasing specific angular momentum are more stable. The algorithm of the numerical experiment proceeds as follows. At the initial step the mass of the black hole was increased by a small amount, typically by about few percent of central object mass. After the time step δt, the elementary mass δM and angular momentum $\delta L = l(R_\mathrm{in})\,\delta M$ are accreted across the horizon, $r = r_+ \equiv [1 + \sqrt{1-a^2}]\,GM/c^2$. The mass increase δM is computed as a difference of the mass of torus $M_\mathrm{d} = \int_\mathcal{V} \rho\,\mathrm{d}\mathcal{V}$ at t and $t + \delta t$, where $\mathrm{d}\mathcal{V} = u^t\sqrt{-g}\,\mathrm{d}^3x$ is taken over the spatial volume occupied by the torus. The corresponding elementary spin increase is $\delta a = l\,\delta M/(M + \delta M)$. Therefore, at each step of the simulation we updated the model parameters by the corresponding low values of mass and angular momentum changes: $M \to M + \delta M$, $a \to a + \delta a$. The inner cusp moves accordingly.

Figure 1 shows the dependence of the torus mass on time for different values β of the ratio between thermodynamical and magnetic pressure (plasma parameter), $\beta \equiv P_\mathrm{g}/P_\mathrm{m}$, for a torus with the radially increasing distribution of angular momentum, $l(R) = l_{\mathrm{K},\,R=R_\mathrm{in}}[1 + \epsilon(R - R_\mathrm{in})]^q$ with $q > 0$, $0 < \epsilon \ll 1$. This means that the reference level of the specific angular momentum is set to $l = \mathrm{const} = l_\mathrm{K}(R_\mathrm{in})$, motivated by the standard theory of thick accretion discs, where the constant value is a limit for stability. A radially growing profile then helps to stabilise the configuration. From the graph we see that the amount of accreted mass is generally larger for smaller β. The plot also shows that the overall gradually decreasing trend is superposed with fast oscillations. After the initial drop of the torus mass (given by the magnitude of the initial perturbation, $\delta M \simeq 0.01\,M$) phases of enhanced accretion change with phases of diminished or zero accretion.

Figure 2 compares the magnetized vs. non-magnetized tori for the same spin ($a = 0.3$). In the top panel we show the time dependence of the radial coordinate of the point with the highest mass density $R = R_\mathrm{c}$ (hence the highest pressure) of these two tori, and in the bottom panel the dependence of the highest mass density is captured as a function of time. In the limit of a non-magnetised slender torus ($R_\mathrm{c} \gg 1$) these oscillations correspond to the situation that has been treated previously by analytical methods (Blaes et al., 2006). Although the amplitude of R_c oscillations is quite small in these examples (because the oscillations were initiated by a weak perturbation and the torus centre is relatively far from the black hole), the outer layers of the torus are affected more significantly and can be accreted across the inner edge.

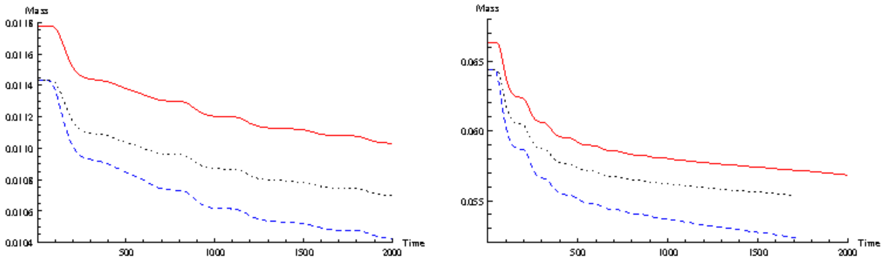

Figure 1. Torus mass, $M_{\rm d}(t)$, relative to the black-hole mass as a function of time. The initial rapid accretion rate results in a drop of $M_{\rm d}$ that becomes partially stabilised during the subsequent evolution. Time is given in dimensionless units of GM/c^3. The orbital period is close to its Keplerian value near the inner edge, i.e. about $\Delta t(R) \simeq 100$ for the material near $R = R_{\rm in}$. *Left panel*: The case of spin $a = 0.1$ is shown for different values of magnetisation parameters $\beta = 3$ (*dashed*), $\beta = 80$ (*dotted*), and $\beta \to \infty$ (i.e. a non-magnetized case; solid line). *Right panel*: as above, but for $a = 0.9$. Figure adopted from (Hamerský and Karas, 2013).

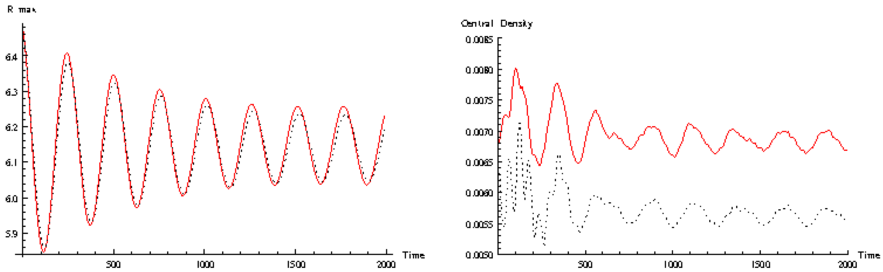

Figure 2. Oscillation of the torus centre $R = R_{\rm c}$ (*left panel*; radius is expressed in geometrized units GM/c^2 on the vertical axis), and of the central density $\rho = \rho_{\rm c}$ (*right panel*); density is relative to its peak value at the centre, $\rho_{\rm c} = \rho(R_{\rm c})$. The *solid* line is for a non-magnetized case ($\beta \gg 1$), the *dotted* line denotes the magnetized configuration ($\beta = 3$). Figure adopted from (Hamerský and Karas, 2013).

2.2 Triggered oscillations and destruction of the torus

Vertical and radial oscillations of accretion tori may cause an overflow of matter onto the black hole because during these oscillations some parts of the torus occurs outside the equipotential surface for the initially stable torus. We assumed a following scenario of the periodically perturbed torus. At the beginning we perturb the distribution of mass inside the torus so that it would oscillate. Then we prescribe additional perturbations in the vertical direction which simulate the crossing of a star through the torus. This scenario is reasonable for intermediate-mass black holes since we can assume that the size of the torus and of the star are of the same order. This situation can be still treated without taking the self-gravity into account. During these crossings of the star mass is dragged up from the equatorial plane and consequently accreted. Before reaching the horizon the material is accelerated to create an outflow. Naturally, the interpretation of star-torus interaction is only tentative, because we use the 2D (azimuthally averaged) scheme.

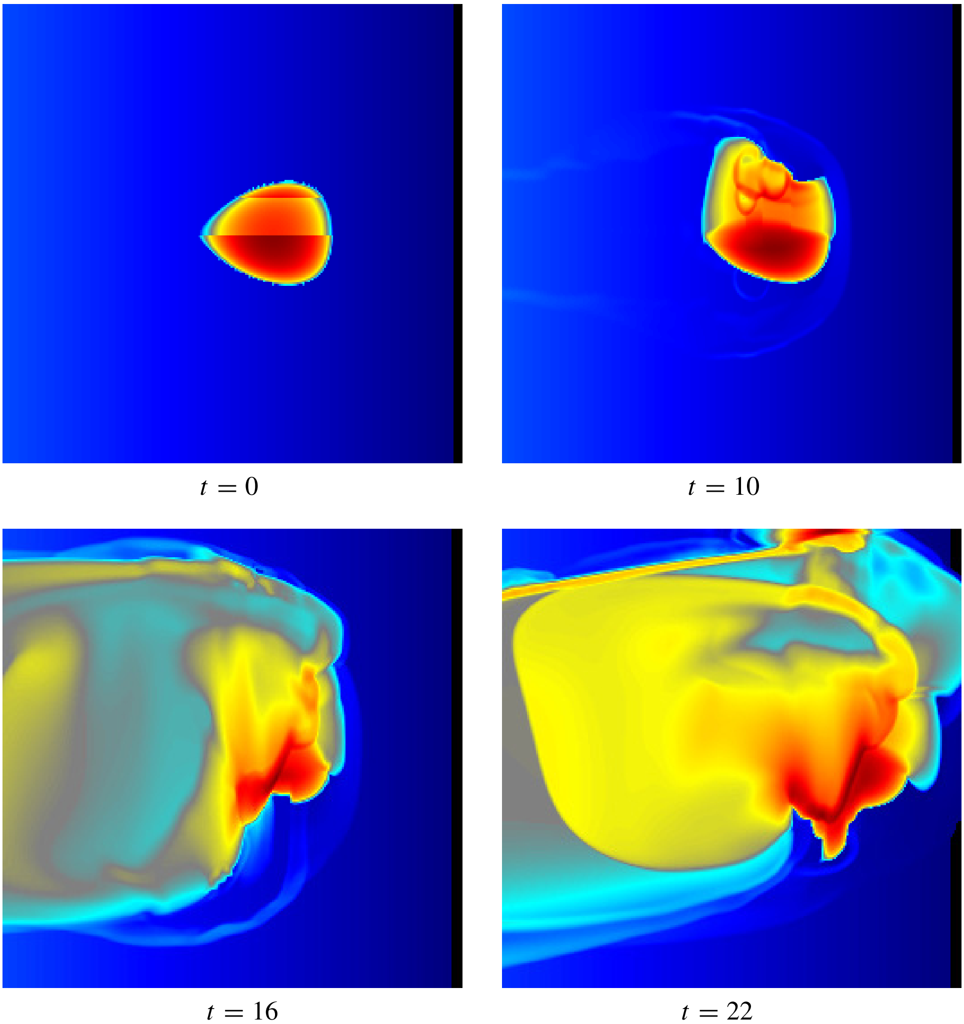

Figure 3. Distribution of mass density in various phases of the simulation (the colour scale in arbitrary units normalized to the maximum density). On the *left-top* panel there is a perturbed distribution of mass at time $t = 0$. *Next* panels show the profile of the torus after increasing the number of "star crossings". Coordinates on the axes are identical to coordinates defined in HARM 2D numerical scheme (Gammie et al., 2003). Geometrical units are used, where time is scaled by GM/c^3.

Figure 3 shows the time evolution of the torus with additional perturbations. On these images the regions of constant radius correspond to vertical lines parallel to the vertical axis (HARM coordinates are used). The top and the bottom rim of the image correspond to rotation axis of the central Kerr black hole.

On the top part of the right-bottom panel in Fig. 3 one can see the outflow directed away from the horizon. This image is shown in Cartesian coordinates in Fig. 4 where the arrows express a direction and a magnitude of the velocity of the mass. The velocity of

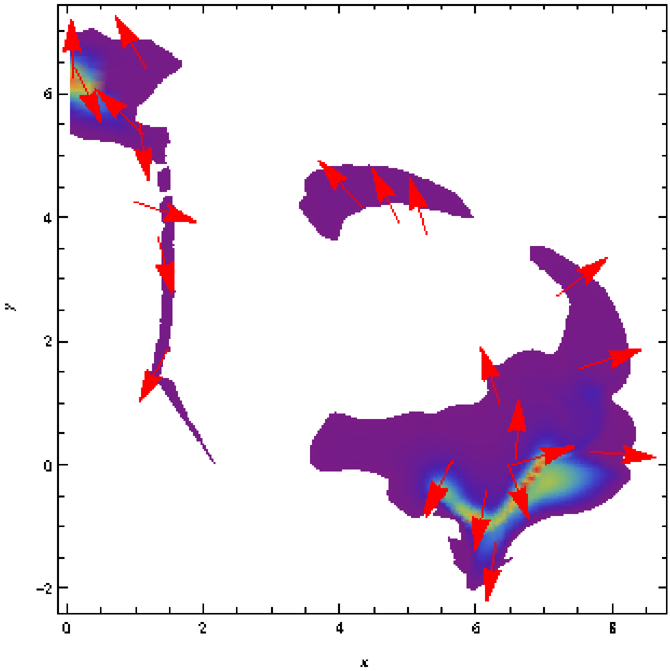

Figure 4. Distribution of mass (colour-coded in arbitrary units) with corresponding velocities indicated by red arrows. While the torus oscillates mainly in the vertical direction, some part of mass high above the equatorial plane has positive radial velocities. (X,Y) are Cartesian coordinates scaled with respect to R_g.

accelerated mass can reach values about $0.3\,c$. We compared the non-magnetized case with the magnetized one. When we assumed the purely toroidal magnetic field present inside the torus according to the Komissarov solution (Komissarov, 2006) with the magnetization parameter $\beta = 3$ there was more mass accreted compared to non-magnetized case and velocities of accelerated mass were higher approximately by 12 %. We also tried to add a vertical component of the magnetic field to our simulations. However the strength of this field was not higher than 10 % of the strength of the toroidal magnetic field because for stronger fields numerical problems in the code arise.

3 CONCLUSIONS

Within the framework of an axially symmetric magnetized fluid torus model we have extended the previous results on the onset of runaway instability of relativistic configurations near a rotating black hole. The numerical approach allows us to consider also large amplitude perturbations that can lead to significant outflows and even the torus destruction. We concentrated on systems with radially increasing specific angular momentum that are threaded by a purely toroidal magnetic field. We neglected self-gravity of the gaseous material (the mass of the torus was set to be at most several percent of the black-hole mass), nevertheless, we allowed for a gradual change of the Kerr metric mass and spin parameters by accretion over the inner edge. The angular momentum distribution within the torus was also allowed to evolve, starting from the initial power-law profile. The mass transfer influences the location of the cusp of the critical configuration, which can lead to the runaway instability. The process of accretion is not perfectly monotonic, instead, there are changing phases of enhanced accretion rate and phases where the mass of torus remains almost constant. The overall gradual decrease of the torus mass is superposed with oscillations that can be seen by following the central density variations on the dynamical time-scale and the position of the centre of the torus. The oscillation amplitude is sensitive to the initial perturbation, but the frequency is not, namely, a small change of the metric coefficients does not affect the oscillation frequency. A large-amplitude perturbation leads to the torus complete destruction.

The toroidal magnetic field plays a more important role in the early phases of the accretion process until the perturbed configuration finds a new equilibrium or disappears because of the runaway instability. We showed that additional perturbations in the vertical direction can lead to relativistic outflows if the perturbations are strong enough so that the mass could get far from the equatorial plane. Otherwise the mass is accelerated dominantly in the radial direction and it moves back to the torus. We showed that the presence of magnetic field supports the acceleration of mass and consequently outflows can reach higher velocities. For further details see Hamerský and Karas (2014, in preparation).

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