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Effects of environmental drag onto a fast-moving magnetic compact star near a supermassive black hole

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ABSTRACT
The hydrodynamical drag by interstellar gaseous environment can influence the orbital motion of stars near a supermassive black hole on long time-scales, even if the medium is diluted. This effect is generally more important for bodies with a large geometrical cross-sectional area, such as supergiants interacting with a relatively dense accretion disc in active galactic nuclei, whereas it is entirely negligible for compact stars embedded in a rarefied accretion flow in low-luminosity galactic nuclei.

We discuss whether a strong magnetic field of a neutron star can significantly enhance the drag effect by increasing the effective cross-sectional area for the mutual interaction, especially in the case of the hypersonic motion. We find that the increase due to magnetic forces is still far too small to be important, e.g. for the long term orbital evolution of the putative population of neutron stars in the Galactic centre, where the environment density is very low.

Keywords: Accretion: accretion discs – black-hole physics

1 INTRODUCTION

The problem of accretion onto a star or a black hole in the presence of magnetic fields has been investigated since many years: for the original account and basic ideas relevant to our present discussion, see e.g. Bisnovatyi-Kogan and Ruzmaikin (1974, 1976); Ghosh and Lamb (1978); Kluźniak and Rappaport (2007). Most attention has been focused towards the accretion fed via an accretion disk. It has been recognized that the accretion process can proceed in various modes, depending mainly on the star compactness and rotation period, the accretion rate, and the magnetic field strength and orientation. In the past, somewhat less attention has been focused towards the magnetic version of the Bondi–Hoyle–Lyttleton problem of accretion onto isolated stars moving rapidly through the interstellar medium (e.g. Toropina et al., 2001, 2006).

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The hydrodynamical drag can be safely ignored in late stages of a compact star inspiraling in a standard disc; see Karas and Šubr (2001). In that paper we considered also giant stars for which the drag is more important simply because of much larger geometrical cross-sectional area for the direct interaction with the disc material. Neutron stars and stellar-mass black holes were suspected to be too tiny and very weakly interacting in this respect (Narayan, 2000).

However, the effective cross section of a *magnetized* neutron star can be significantly larger and it can lead to an increased drag. We thus summarize formulae describing the interaction of a rotating magnetic star with accretion plasma (for a textbook account of the subject, see Lipunov (1987); also Romanova et al. (2005)).

2 BASIC ESTIMATES BASED ON CRITICAL RADII

Accretion of material by a star or a compact object has been studied since the late 1930s. First, astrophysicists investigated axially symmetric accretion onto a star moving through a cloud of interstellar medium (see "The evolution of stars" by Hoyle and Lyttleton (1939) and "On the effect of interstellar matter on the motion of a star" by Dodd and McCrea (1952)). A possibility was proposed of terrestrial climatic effects being due to density variations of the medium in the solar neighbourhood. Next, in the 1950s, the theory of spherical accretion of gas was developed (Bondi, 1952, "On spherically symmetric accretion"). This line of research has continued with more complicated studies of gas transport between individual components in binary systems, and led to the idea of accretion disks in the late 1960s.

A trivial order-of-magnitude estimate of the gravitational potential energy which can in principle be released in the course of accretion of a test mass m onto a spherical body with mass M and radius R_* gives

$$\Delta E_{\rm acc} = \frac{GMm}{R_*} \begin{cases} \overset{\dagger}{\approx} 10^{20}\, \frac{M}{M_\odot}\, \frac{m}{1\,{\rm g}}\, \frac{10\,{\rm km}}{R_*} \quad [{\rm erg}]\,, \\ \overset{\ddagger}{\approx} 10^{53}\, \frac{M}{10^8\,M_\odot}\, \frac{m}{M_\odot}\, \frac{10^{-4}\,{\rm pc}}{R_*} \quad [{\rm erg}]\,. \end{cases} \tag{1}$$

Typical values for a neutron star (†) and for a super-massive black hole (‡) have been used in numerical estimates. Let us compare $\Delta E_{\rm acc}$ with the energy which could be extracted from the same mass m by nuclear fusion reactions. Hydrogen-to-helium burning, the most important case from the astrophysical viewpoint, gives

$$\Delta E_{\rm nuc} = \Delta m c^2 \begin{cases} \approx\; 5 \times 10^{18}\, \frac{m}{1\,{\rm g}} \quad [{\rm erg}] \overset{\dagger}{\approx} 0.1\, \Delta E_{\rm acc}\,, \\ \approx\; 10^{52}\, \frac{m}{M_\odot} \quad [{\rm erg}] \quad \overset{\ddagger}{\approx} 0.1\, \Delta E_{\rm acc}\,. \end{cases} \tag{2}$$

Energy potentially releasable by accretion is very sensitive to a dimension-less *compactness parameter*,

$$\varepsilon \equiv \frac{2GM}{R_* c^2}\,. \tag{3}$$

Order-of-magnitude estimates of parameter ε: (i) Neutron stars – $R_* \approx 10$ km, $\varepsilon \approx 0.1$; (ii) White dwarfs – $R_* \approx 10^4$ km, $\varepsilon \approx 10^{-4}$ (as an example we mention binary systems consisting of a white dwarf which accretes matter from a close, usually main sequence companion; these are cataclysmic variables); (iii) Solar-type stars – $R_* \approx 10^6$ km, $\varepsilon \approx 10^{-6}$; for example standard main-sequence stars in a binary system (symbiotic stars) belong to this category; (iv) Black holes – $R_* \approx R_g \equiv 2GM/c^2 \approx 3(M/M_\odot)$ km, $\varepsilon \gtrsim 0.1$ (black holes have no rigid surface. Under suitable conditions, R_* coincides with the last, innermost stable orbit below which material falls freely into the black hole. In the case of a non-rotating, Schwarzschild black hole, $R_* = 3\,R_g$, $\varepsilon = 1/3$.)

Another dimensionless quantity is also frequently designated as the compactness parameter in the theory of accretion onto compact objects. It takes into account the radiation luminosity L of the object:

$$\tilde{\varepsilon} \equiv \frac{L\,\Xi_T}{R_* m_e c^3}\,. \tag{4}$$

(The Thomson cross-section for electrons is $\Xi_T = 6.65\,246 \times 10^{-25}$ cm^2.)

In many situations, the accreted matter has apparently a non-negligible value of angular momentum which invalidates the basic assumption of the spherical approximation. Non-spherical accretion was originally investigated in the case of an interstellar medium captured by a moving object, e.g. a star in a nebula (Bondi and Hoyle, 1944).

Let us consider a *magnetic* star moving as a “bullet” along an inclined trajectory across the accretion disc plane. In the case of supersonic motion, the relevant interaction radius is called the Shvartsman radius (e.g. Lipunov, 1987; Romanova et al., 2001),

$$R_{\rm Sh} \simeq 9.4 \times 10^{15} B_{12} P_1^{-2} v_7^{-1} n^{-1/2}\,[\rm cm]\,. \tag{5}$$

Notation for relevant variables follows the standard practice (e.g. Lipunov, 1987), in particular, B_{12} denotes the magnetic intensity scaled to the units of 10^{12} Gauss.

Here one assumes that two conditions, $R_{\rm Sh} > R_{\rm acc}$ and $R_{\rm Sh} > R_{\rm lc}$, are both satisfied, where the light cylinder is

$$R_{\rm lc} = c/\Omega_* \simeq 4.8 \times 10^{12} P_3 [\rm cm]\,, \tag{6}$$

and the Bondi–Hoyle radius (Edgar 2004) is

$$R_{\rm acc} = \frac{2GM}{w^2} = 9.4 \times 10^{11} \frac{M_{1.4}}{w_{200}^2}\,[\rm cm]\,, \tag{7}$$

$$w_{200} = \frac{\sqrt{c_s^2 + v^2}}{200\,\rm km/s} \tag{8}$$

(c_s is the sound speed, v is relative velocity).

The above given formula for $R_{\rm Sh}$ follows from the equality between the magnetic pressure P_m (due to a rotating dipole luminosity L_m),

$$P_m = \frac{L_m}{4\pi R^2 c} = \frac{\kappa_t \mu^2}{4\pi R_{\rm lc}^4 R^2} \tag{9}$$

($\kappa_t \sim 1/2$), and the ram pressure

$$P_{\rm ram} = \rho v^2 \simeq \frac{\dot{M} v^5}{G^2 M^2}\,. \tag{10}$$

Alternatively, one can write

$$R^2_{\rm Sh} = \frac{\kappa_t \mu^2 \Omega_*^4 G^2 M^2}{2\dot{M} v^5 c^4}\,. \tag{11}$$

In these circumstances the accretion rate is small (much less than what would correspond to the direct accretion onto $R_{\rm Sh} > R_{\rm lc} \gg R_*$ sphere), but the momentum exchange can still be significantly large, causing possibly a non-negligible drag.

On the other hand, in case of a star embedded in the disc plane, the relative velocity is small with respect to the surrounding medium, and so the characteristic cross-section is now given by the Alfvén (stopping) radius, $R_{\rm A}$. For the latter one can derive different formulae depending on the exact situation:

Furthermore,

$$R_{\rm Ag} = \left(\frac{\kappa_g \mu^2}{\dot{M}_{\rm BHL}}\right)^{2/7}\,, \tag{12}$$

where $\kappa_g \sim (2GM)^{-1/2}/2$ is a constant, $\mu = B_* R_*^3/2$ is the magnetic moment. The asterisk denotes quantities that correspond to the star surface (in case of a dipole field on finds B_* at the pole $= 2B_*$ at equator; the numerical factor is somewhat uncertain in κ_g).

The Bondi–Hoyle–Lyttleton accretion rate (BHL, assumed in the above-given relation) is

$$\dot{M}_{\rm BHL} = \frac{\sigma \pi G^2 M^2 \rho_\infty}{w^3} = \pi R^2_{\rm acc} w \rho_\infty/2\,, \tag{13}$$

$\sigma \sim 4$. The equation for $R_{\rm Ag}$ is relevant in the situation when $R_{\rm Ag} < R_{\rm acc}$ and $R_{\rm Ag} < R_{\rm lc}$ both hold.

Finally,

$$R_{\rm Ap} = \left(\frac{\kappa_p \mu^2 G^2 M^2}{\dot{M}_{\rm BHL} w^5}\right)^{1/6}\,, \tag{14}$$

$\kappa_p \sim 1$ or 2. The latter relation for $R_{\rm Ap}$ is relevant when $R_{\rm Ap} > R_{\rm acc}$ and $R_{\rm Ap} < R_{\rm lc}$. In this case one can expect the accretion rate to be given roughly by $\dot{M}_{\rm BH}$, but the momentum exchange is smaller than it was in the previous case (however, it takes continuously, while the star is embedded with the disc medium).

The above given derivation contains various parameters evaluated at the star surface – apart from the magnetic intensity B_* (or the magnetic dipole moment μ), it is the linear velocity of the star motion v, radius of the star R_*, the mass M, and the period of rotation P. It also contains the density of the environment ρ (resp. n), and the corresponding velocity of sound, $c_{\rm s}$.

In order to derive specific conclusions about the drag efficiency, one has to fix several parameters at their typical values or to set the typical values expected in the accretion disk. Only then one is prepared to investigate the dependency on B_*, which is the relevant information that we seek.

The magnetic field depends on the rotation period, so it is interesting to examine also the time dependence $\Omega_*(t)$. The most prominent deceleration effect of this kind is expected to occur in case of magnetars (e.g. Toropina et al., 2006) – they slow down on the time-scale of 10^3–10^4 yrs. One can involve the power-law deceleration according to Mori and Ruderman (2003):

$$I\dot{\Omega} = -\kappa \mu^{n_1} \rho^{n_2} v^{n_3} \Omega^n \,, \tag{15}$$

where $n_1 = (3+n)/3$, $n_2 = (3-n)/6$, $n_3 = (3-4n)/3$, $\kappa = \text{const}$, $n = \text{const}$ $(-1 < n < 2)$.

We can thus conclude that the magnetic star effective radius can be much larger than the geometrical one. However, the characteristic radii do not capture the entire reality; they only partly reflect the basic operation of the drag, which needs to be confirmed by computations carried out under more realistic assumptions. Obviously, the astrophysically realistic modelling needs MHD numerical simulations, such as those described by, e.g. Romanova et al. (2003); Spitkovsky (2006); Toropina et al. (2008), where the complex structure of the magnetosphere can be properly modelled.

3 CONCLUSIONS

The drag force is exerted on a moving star by the ambient medium. We can roughly estimate the effect by the cross-sectional area for the mutual interaction between the moving body and the surrounding interstellar gas. Compared with the geometrical radius, the magnetic "stand-off" radius (given by the equilibrium between the magnetic pressure and the hydrodynamic pressure) is significantly larger; it is the greatest one of several characteristic radii. One can expect that the stand-off radius determines the magnitude of the total drag force. To see this more clearly we listed different characteristic radii of the problem; these depend on the type of drag force that one takes into account (due to the thermal pressure versus the ram pressure acting against the star linear motion), and how quickly the magnetic effects decay with the distance from the star (dipole field versus the radiation field of a fast rotator).

Under realistic conditions of the supermassive black hole in the Galactic centre and similar low-luminosity (highly sub-Eddington) nuclei, however, the order-of-magnitude estimation of the characteristic radii for the interaction as well as numerical experiments show that the magnetic field of a compact star does not enhance the drag force significantly (Karas and Šubr, 2001), and so the orbital mechanics of magnetic compact stars comes out only slightly altered in comparison with the non-magnetised case. This conclusion ensures that the drag forces on compact stars can be safely neglected in calculations of orbital evolution of the neutron-star population near the Galactic centre.

Nonetheless, the interaction is still relevant in the context of creation of bow-shock structures (van Marle et al., 2011; Meyer et al., 2014). These have been revealed in several cases also in the Galactic centre (Mužić et al., 2010), where a dense cluster of fast-moving stars on close orbits exists and should include the tentative population of compact magnetized objects close to the supermassive black hole (Zajaček et al., 2014).

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