Oscillations of electric current-carrying string loop near a Schwarzschild black hole immersed in an asymptotically uniform magnetic field

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ABSTRACT
We study oscillations of an electric current-carrying and axially-symmetric string loop in the vicinity of a Schwarzschild black hole embedded in an asymptotically uniform magnetic field. The radial profiles of frequencies of small oscillations of the string loop around stable equilibrium points are given for the radial and vertical harmonic modes that are relevant also in the quasi-periodic stages of the oscillations. Their properties in dependence on the uniform magnetic field intensity and angular momentum parameters of the string loops are determined. We examine the relevance of resonant phenomena of the radial and vertical string-loop oscillations at their frequency ratio 3:2. The oscillatory frequencies of the string loops are compared with the frequencies of high-frequency quasi-periodic oscillations (HF QPOs) observed in the microquasars GRS 1915+105, XTE 1550-564, GRO 1655-40 containing a black hole. We have demonstrated that the influence of the uniform magnetic field does not allow us to explain all the observed data for non-rotating black holes. Clearly, rotation of the black hole is necessary to explain the all the observed frequencies in the microquasars by the string loop oscillations.

Keywords: string loop oscillations – X-ray variability – HF QPO observations

1 INTRODUCTION

Relativistic current-carrying string loops moving axisymmetrically along the symmetry axis of the Kerr or Schwarzschild–de Sitter black holes have been recently studied extensively (Jacobson and Sotiriou, 2009; Kološ and Stuchlík, 2010, 2013). Such a configuration was also studied in (Larsen, 1994; Frolov and Larsen, 1999). Tension of such string loops prevents their expansion beyond some radius, while their worldsheet current introduces an angular momentum barrier preventing them from collapsing into the black hole. There is an important possible astrophysical relevance of the current-carrying string loops (Jacobson and Sotiriou, 2009), as they could in a simplified way represent plasma that exhibits

978-80-7510-127-3

associated string-like behaviour via dynamics of the magnetic field lines in the plasma (Christensson and Hindmarsh, 1999; Semenov et al., 2004), or due to thin isolated flux tubes of magnetized plasma that could be described by an one-dimensional string (Spruit, 1981; Semenov and Bernikov, 1991; Cremaschini and Stuchlík, 2013). Motion of electrically charged string loops in combined external gravitational and electromagnetic fields has been recently studied for a Schwarzschild black hole immersed in a homogeneous magnetic field (Tursunov et al., 2013, 2014).

Understanding of the dynamics of charged particles in the combined electromagnetic and gravitational fields is necessary for the modelling of the MHD processes. The single-particle dynamics is relevant also for collective processes modelled in the framework of kinetic theory (Cremaschini and Stuchlík, 2013; Cremaschini et al., 2013; Cremaschini and Stuchlík, 2014). The oscillatory motion of charged particles around equatorial and off-equatorial circular orbits could be relevant in formation of magnetized string loops (Cremaschini and Stuchlík, 2013; Kovář, 2013). The string-like configurations of magnetized plasmas could occur in the accretion discs due to an instability or irradiation creating an ansamble of charged particles in epicyclic motion giving rise to the stringy structure due to kinetic dynamo effect. A nearly uniform and stable magnetic field can be naturally generated by a distant magnetar Kovář et al. (2014) a strongly magnetized star.

The astrophysical applications of the current carrying string loops have been focused on the problem of acceleration of string loops due to the transmutation process (Jacobson and Sotiriou, 2009). Since the string loops can be accelerated to ultra-relativistic velocities in the deep gravitational potential of compact objects (Stuchlík and Kološ, 2012a,b), the string loop transmutation can be well considered as a process of formation of ultra-relativistic jets, along with the standard model based on the Blandford–Znajek process (Blandford and Znajek, 1977). Here we concentrate out attention on the inverse situation of small oscillations of string loops in the vicinity of stable equilibrium points at the equatorial plane of black holes that was proposed as a possible model of HF QPOs observed in black hole and neutron star binary systems (Stuchlík and Kološ, 2012b).

In the black hole systems observed in both Galactic and extragalactic sources, strong gravity effects have a crucial role in three phenomena related to the accretion disc that is the emitting source: the spectral continuum, spectral profiled lines, and oscillations of the disc; clearly, strong gravity has an important role also in the binary systems containing neutron (quark) stars. HF QPOs of X-ray brightness had been observed in many Galactic Low Mass X-Ray Binaries (LMXB) containing neutron stars (see e.g. van der Klis, 2000; Barret et al., 2005; Belloni et al., 2007) or black holes (see e.g. McClintock and Remillard, 2006; Remillard, 2005; Remillard and McClintock, 2006). Some of the HF QPOs are in the kHz range and often come in pairs of the upper and lower frequencies ($\nu_{\rm U}$, $\nu_{\rm L}$) of *twin peaks* in the Fourier power spectra. Since the peaks of high frequencies are close to the orbital frequency of the marginally stable circular orbit representing the inner edge of Keplerian discs orbiting black holes (or neutron stars), the strong gravity effects must be relevant in explaining of HF QPOs (Török et al., 2005).

It has been shown in (Stuchlík and Kološ, 2014) that the frequencies of the twin peak oscillations observed in spectra of three different microquasars can be explained by the oscillations of string loop in the field of a Kerr black hole. Here we aim to extend previous research to the case of Schwarzschild black hole immersed in external uniform magnetic field. Assuming small oscillations of a string loop near an equilibrium position corresponding to a minimum of the effective potential, the Hamiltonian of string loop motion can be perturbed with the first order term corresponding to linear harmonic oscillators in two uncoupled radial and vertical orthogonal modes (Stuchlík and Kološ, 2014). The higher order terms correspond to the non-linear phenomena causing coupling of the radial and vertical oscillatory modes and determine transition to chaotic motion through quasi-periodic stages of the oscillatory motion. The frequencies of the radial and vertical harmonic oscillations are relevant also in the quasi-periodic stages of the oscillatory motion (Kološ and Stuchlík, 2013).

2 MODEL OF STRING LOOP OSCILLATIONS

We study a string loop motion in the field of a black hole described by the Schwarzschild metric, characterized by the gravitational mass M,

$$\mathrm{d}s^2 = -A(r)\,\mathrm{d}t^2 + A^{-1}(r)\,\mathrm{d}r^2 + r^2\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\big)\,, \quad A(r) = 1 - \frac{2M}{r}\,. \tag{1}$$

We use the geometric units with $c = G = 1$ and the Schwarzschild coordinates. In order to properly describe the string loop motion, it is useful to use the Cartesian coordinates

$$x = r\sin(\theta)\,, \quad y = r\cos(\theta)\,. \tag{2}$$

The string loop is threaded onto an axis of the black hole chosen to be the y-axis. Due to the assumed axisymmetry of the string loop motion, one point path can represent whole movement of the string. Trajectory of the string can be represented by a curve in the 2D x-y plane. The string loop can oscillate, changing its radius in the x-z plane, while propagating in the y direction.

We assume static, axisymmetric and asymptotically uniform magnetic field. Since the Schwarzschild spacetime is flat at spatial infinity only nonzero covariant component of the potential of the electromagnetic field takes the form (Wald, 1974)

$$A_\phi = \frac{B}{2}r^2\sin^2\theta = \frac{B}{2}x^2\,. \tag{3}$$

The symmetries of the considered background gravitational and magnetic fields, corresponding to the t and ϕ components of the Killing vector, imply the existence of two constants of the motion, namely the string loop energy E and the string loop angular momentum L (Tursunov et al., 2013, 2014).

Dynamics of an axisymmetric current-carrying string loop in a given axially symmetric and stationary Kerr spacetime in the absence of electromagnetic fields has been discussed in detail in (Jacobson and Sotiriou, 2009; Kološ and Stuchlík, 2013; Stuchlík and Kološ, 2014). In the spherically symmetric spacetime (1) immersed in external magnetic field the Hamiltonian governing the string loop dynamics can be expressed in the form (Tursunov et al., 2013)

$$H = \frac{1}{2}f(r)P_r^2 + \frac{1}{2r^2}P_\theta^2 - \frac{E^2}{2f(r)} + \frac{V_{\mathrm{eff}}}{2f(r)}\,, \tag{4}$$

with an effective potential for the string loop motion in the combined gravitational and magnetic fields

$$V_{\rm eff} = f(r)\left\{\frac{B^2x^3}{8} + \left(\frac{\Omega J B}{\sqrt{2}} + \mu\right)x + \frac{J^2}{x}\right\}^2 . \tag{5}$$

In accordance with (Jacobson and Sotiriou, 2009), we have introduced new parameters that are conserved during the motion of string loop in the Schwarzschild spacetime combined with the uniform magnetic field,

$$J^2 \equiv \frac{j_\sigma^2 + j_\tau^2}{2}, \quad \omega \equiv -\frac{j_\sigma}{j_\tau}, \quad \Omega \equiv \frac{-\omega}{\sqrt{1+\omega^2}}, \tag{6}$$

where the parameters j_τ, j_σ determines current of the string. The parameter J is always positive, $J > 0$, the dimensionless parameter ω runs in the interval $-\infty < \omega < \infty$, and the dimensionless parameter Ω varies in the range $-1 < \Omega < 1$ (Tursunov et al., 2013, 2014).

We shall use for simplicity the dimensionless radial coordinate $r/M \to r$, dimensionless time coordinate $t/M \to t$, and we make the rescaling $E/\mu \to E$ and $J/\sqrt{\mu} \to J$.

The equations of motion for $\mu \in \{r, \theta\}$ are given by the Hamilton equations relating the position 4-vector and 4-momentum of the string loop

$$\frac{\mathrm{d}X^\mu}{\mathrm{d}\zeta} = \frac{\partial H}{\partial P_\mu}, \quad \frac{\mathrm{d}P_\mu}{\mathrm{d}\zeta} = -\frac{\partial H}{\partial X^\mu}. \tag{7}$$

The properties of the effective potential $V_{\rm eff}(r,\theta)$, (5) were discussed in great details in (Tursunov et al., 2013, 2014), here we give a short overview. The local extrema of the effective potential cannot be located out of the equatorial plane corresponding to $y = 0$. Then the extrema of the angular momentum parameter of the string loop correspond to

$$J = J_{\rm E\pm}(x; B, \Omega) \equiv \frac{B\Omega x^2(x-1) \mp \sqrt{G}}{2\sqrt{2}(x-3)} \tag{8}$$

where

$$G(x; \Omega, B) = B^2(x-1)^2x^2\Omega^2 + B^2(x-3)(3x-5)x^2 + 8(x-3)(x-1) . \tag{9}$$

The behaviour of the functions $J_{\rm E\mp}(x; B, \Omega)$ is discussed in detail in (Tursunov et al., 2013).

There are four different types of the boundaries for string loop motion given by the condition

$$V_{\rm eff}(x,y) = E^2 = \text{const.}, \tag{10}$$

for the string loop dynamics in the background constituted by a Schwarzschild BH immersed in an uniform magnetic field. We can distinguish them according to two properties: possibility of the string loop to escape to infinity in the y-direction, and possibility to collapse to the black hole. A detailed discussion can be found in (Kološ and Stuchlík, 2010; Tursunov

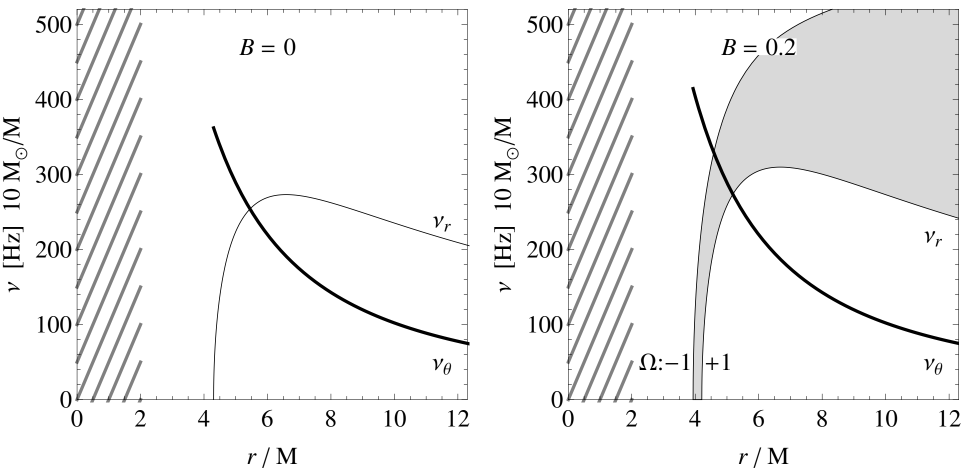

Figure 1. String-loop oscillatory frequencies ν_{r} (*thin curves*) and ν_θ (*thick curves*), calculated in case of the Schwarzschild black hole with mass $M = 10\mathrm{M}_\odot$ for the absence (*left plot*) and the presence (*right plot*) of external magnetic field. We demonstrate extension of the frequency radial profiles for the complete range of the string loop parameter $\varOmega \in \langle -1, 1\rangle$ for $B = 0.2$ case (*greyed area*). The $B = 0$ case is independent of the string loop parameter $\varOmega$. Due to the symmetry of the uniform magnetic field, the vertical frequency ν_θ is independent of the parameter B. The area inside the horizon is *dashed*.

et al., 2013). The first case corresponds to no inner and outer boundary – the string loop can be captured by the black hole or escape to infinity. The second case corresponds to the situation with an outer boundary – the string loop must be captured by the black hole. The third case corresponds to the situation when both inner and outer boundary exist – the string loop is trapped in some region forming a potential "lake" around the black hole. The fourth case corresponds to an inner boundary – the string loop cannot fall into the black hole but it must escape to infinity. For our following discussion only the third case, corresponding to the possibility of the string loop to be trapped in some region, will be relevant.

2.1 Frequency of the radial and vertical harmonic oscillatory modes

The Hamiltonian (4) can be written as a sum of the dynamic and potential parts

$$H = H_{\mathrm{D}} + H_{\mathrm{P}} = \frac{1}{2} g^{rr} P_r^2 + \frac{1}{2} g^{\theta\theta} P_\theta^2 + H_{\mathrm{P}}(r, \theta) \,. \tag{11}$$

The string loop harmonic oscillations around a stable equilibrium position with fixed coordinates r_0 and $\theta_0 = \pi/2$ have the locally measured angular frequencies of the radial and vertical oscillatory motion given by (Stuchlík and Kološ, 2014)

$$\omega_{\mathrm{r}}^2 = \frac{1}{g_{rr}} \frac{\partial^2 H_{\mathrm{P}}}{\partial r^2} \,, \quad \omega_\theta^2 = \frac{1}{g_{\theta\theta}} \frac{\partial^2 H_{\mathrm{P}}}{\partial \theta^2} \,. \tag{12}$$

The partial derivatives of the potential part of the Hamiltonian are calculated at the local minimum of the energy boundary function (effective potential) at r_0 and $\theta_0 = \pi/2$ which is governed by the angular momentum parameter J of the string loop.

The locally measured angular frequencies are connected with the angular frequencies measured by a distant observer, $\Omega_{(\mathrm{r},\theta)}$, by the gravitational redshift transformation (Stuchlík and Kološ, 2014) has the form

$$\Omega_{(\mathrm{r},\theta)} = \frac{\omega_{(\mathrm{r},\theta)}}{P^t} \,. \tag{13}$$

If the angular frequencies $\Omega_{(\mathrm{r},\theta)}$, or frequencies $\nu_{(\mathrm{r},\theta)}$, are expressed in the physical units, their dimensionless form has to be extended by the factor c^3/GM. Then the frequencies of the string loop oscillations measured by the distant observers are given by

$$\nu_{(\mathrm{r},\theta)} = \frac{1}{2\pi}\frac{c^3}{GM}\,\Omega_{(r,\theta)} \,. \tag{14}$$

This is the same factor as the one occurring in the case of the orbital and epicyclic frequencies of the geodesic motion in the Kerr spacetime (Aliev and Galtsov, 1981; Török and Stuchlík, 2005; Stuchlík and Schee, 2012). The order of magnitude and the mass-scaling of the frequencies of the radial and vertical oscillations are the same for both the current-carrying string loops and test particles and one can expect that the string loop oscillations could serve as an explanation of the HF QPOs observed in the strong gravity regions of black holes and neutron stars. The angular frequencies of the string loop oscillations related to a distant observer take the following dimensionless form

$$\begin{aligned}\Omega_{\mathrm{r}}^2(r;\Omega,B) =& \frac{1}{r^4\left(B^2r^4+4\sqrt{2}BJ_{\mathrm{E}}r^2\Omega+8J_{\mathrm{E}}^2+8r^2\right)^2}\\ &\times\Big[16J_{\mathrm{E}}^2r^3\Big(B^2r\left(r^2-6r+4\right)\left(2\Omega^2+1\right)-16\Big)+256r^4\\ &+16\sqrt{2}BJ_{\mathrm{E}}r^4\Omega\Big(3B^2r^4-13B^2r^3+4\left(3B^2+1\right)r^2-24r+16\Big)\\ &+r^4\Big(15B^4r^6-62B^4r^5+12B^2\left(5B^2+8\right)r^4-416B^2r^3-384r\Big)\\ &+64r^6\left(6B^2+1\right)-128\sqrt{2}BJ_{\mathrm{E}}^3r^3\Omega+64J_{\mathrm{E}}^4\left(3r^2-14r+12\right)\Big],\end{aligned} \tag{15}$$

$$\Omega_\theta^2(r) = \frac{1}{r^3}\,, \tag{16}$$

where the function $J_{\mathrm{E}}(r;\Omega,B)$ is given by (8). Due to the symmetry of the uniform magnetic field (3), the horizontal frequency Ω_θ^2 is independent of the effect of magnetic field given by the magnetic intensity parameter B and hence also independent of the string parameter Ω.

In the Schwarzschild spacetime without magnetic field, the harmonic oscillations have frequencies (15–16) relative to distant observers given by expressions relatively very simple for both string loops and test particles. In the case of string loops they read (in dimensional form)

$$\Omega_{\mathrm{r}}^2(r) = \frac{3M^2-5Mr+r^2}{r^4}\,, \quad \Omega_\theta^2(r) = \frac{M}{r^3}\,, \tag{17}$$

while for the epicyclic motion of test particles there is

$$\Omega^2_{\mathrm{r(geo)}}(r) = \frac{M(r-6M)}{r^4}, \quad \Omega^2_{\theta(\mathrm{geo})}(r) = \frac{M}{r^3}. \tag{18}$$

It is quite interesting that the latitudinal frequency of the string loop oscillations in the Schwarzschild or other spherically symmetric spacetimes equals to the latitudinal frequency of the epicyclic geodetical motion as observed by distant observers – for details see (Stuchlík and Kološ, 2012b). Therefore, only gravity is responsible for this frequency in both cases.

Dependencies of the radial and vertical frequencies of the string loop harmonic oscillations on the distance from the black hole are illustrated in Fig. 1 for the characteristic values of the magnetic field intensity $B = 0, 0.2$. In the Schwarzschild spacetime without magnetic field both the frequencies are independent on the parameter Ω, see Fig. 1 (left). In the Schwarzschild spacetime with magnetic field B, the range of the radial and vertical frequencies depends on the string-loop parameter Ω, and the parameter B of the magnetic field. Clearly, the range of allowed frequencies increases with increasing the strength of magnetic field B for the full range of the angular momentum parameter Ω, see Fig. 1 (right).

3 TWIN HF QPOS IN BLACK HOLE SOURCES

The quasi-periodic character of the motion of string loops trapped in a toroidal space around the equatorial plane of a Schwarzschild black hole suggests interesting astrophysical application related to the HF QPOs observed in binary systems containing a black hole or a neutron star, or in active galactic nuclei. Some of the HF QPOs come in pairs of the upper and lower frequencies (ν_{U}, ν_{L}) of *twin peaks* in the Fourier power spectra. Since the peaks of high frequencies are close to the orbital frequency of the marginally stable circular orbit representing the inner edge of Keplerian discs orbiting black holes (or neutron stars), the strong gravity effects must be relevant in explaining HF QPOs (Török et al., 2005). Usually, the Keplerian orbital and epicyclic (radial and latitudinal) frequencies of geodetical circular motion (Török and Stuchlík, 2005; Kotrlová et al., 2008; Stuchlík and Kotrlová, 2009) are assumed in models explaining the HF QPOs in both black hole and neutron star systems.

Before the twin peak HF QPOs have been discovered in microquasars (first by Strohmayer, 2001), and the 3:2 ratio pointed out, (Kluzniak and Abramowicz, 2001) suggested on theoretical grounds that these QPOs should have rational ratios, because of the resonances in oscillations of nearly Keplerian accretion disks; see also (Aliev and Galtsov, 1981). It seems that the resonance hypothesis is now well supported by observations, and the 3:2 ratio ($2\nu_{\mathrm{U}} = 3\nu_{\mathrm{L}}$) is seen most often in twin peak QPOs in the LMXB containing black holes (microquasars). Here we concentrate on the case of 3:2 frequency ratio oscillations observed in three microquasars, GRO 1655-40, XTE 1550-564 and GRS 1915+105, that were discussed in recent literature (Török et al., 2011).

Unfortunately, neither of the recently discussed models based on geodesic oscillatory motion is able to explain the HF QPOs in all the microquasars (Török et al., 2011). Therefore, it is of some relevance to let the string loop oscillations, characterized by their radial and vertical (latitudinal) frequencies, to enter the play, as these frequencies

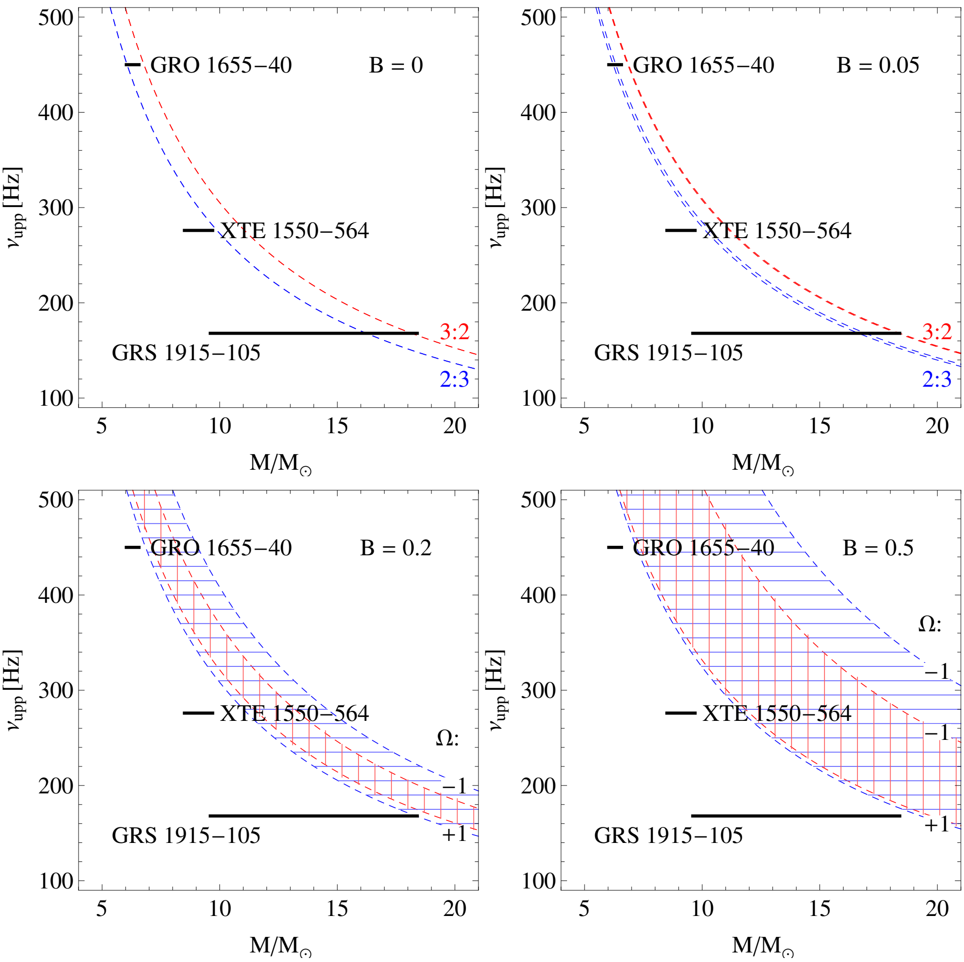

Figure 2. The upper string-loop oscillation frequency ν_U at the 3:2 or 2:3 resonance radii, calculated in the framework of the string-loop model with maximal range of the string-loop parameter Ω as a function of the black hole mass for typical values of magnetic field $B = 0, 0.05, 0.2, 0.5$, and compared to the mass-limits obtained from observations of the three microquasars GRO 1655-40, XTE 1550-564, GRG 1915-105 independent of HF QPO observation and depicted by the horizontal thick lines. Hatched areas cover the whole interval of $\Omega \in \langle -1, 1 \rangle$. The vertical red hatch corresponds to the $3:2$ frequency ratio ν_θ/ν_r, while the horizontal blue hatch corresponds to the 2:3 frequency ratio. The frequency ν_U can appear in both 3:2 or 2:3 resonance radii.

are comparable to the epicyclic geodetical frequencies, but slightly different, enabling thus some relevant corrections to the predictions of the models based on the geodetical epicyclic frequencies ν_θ, ν_r. We again keep the assumption of the resonance phenomena occurring in the oscillatory motion. The resonant phenomena (parametric or forced) are discussed in standard textbooks (Landau and Lifshitz, 1969; Nayfeh and Mook, 1979), discussion of their relevance to the accretion phenomena can be found, e.g. in (Stuchlík et al., 2013).

We can assume applicability of the parametric resonance, discussed in (Landau and Lifshitz, 1969), focusing attention to the case of the frequency ratios $\nu_\theta : \nu_\mathrm{r} = 3{:}2$ or $\nu_\theta : \nu_\mathrm{r} = 2{:}3$, as the observed values of the twin HF QPO frequencies for GRO 1655-40, XTE 1550-564 and GRS 1915+105 sources show clear ratio

$$\nu_\mathrm{U} : \nu_\mathrm{L} = 3 : 2 \tag{19}$$

for the upper ν_U and lower ν_L frequencies. We identify directly the frequencies ν_U, ν_L with ν_θ, ν_r or ν_r, ν_θ frequencies. In contrast to the resonance epicyclic model, the string loop oscillation model allows both frequency ratios

$$\nu_\theta : \nu_\mathrm{r} = 3 : 2 \,, \quad \nu_\theta : \nu_\mathrm{r} = 2 : 3 \,. \tag{20}$$

Since $r_{3:2} < r_{2:3}$, we call the first resonance radius, where $\nu_\theta : \nu_\mathrm{r} = 3{:}2$, the inner one, and the second resonance radius, where $\nu_\theta : \nu_\mathrm{r} = 2{:}3$, the outer one.

For the fixed magnetic field B and fixed string loop parameter ω the upper frequency of the twin HF QPOs can be given as a function of the black hole mass M. If the black hole mass is restricted by independent observations, as is usually the case, we can obtain some restrictions on the string-loop resonant oscillations model, as illustrated in Fig. 2, where the situation is demonstrated for some values of magnetic field $B = 0,\ 0.05,\ 0.2,\ 0.5$. One can see from these plots that the string loop model can well fit the HF QPOs in GRO 1655-40 and GRS 1915-105 sources and gives the limitation on the magnetic field strength. However the observed frequencies for given mass are always lower then the frequencies given by the string loop model. Despite the fact that the parameter Ω widens the frequency range of vertical oscillations, increasing of the magnetic field B implies again increasing frequencies. In other words, for any set of parameters of the model there is no possibility to decrease the frequencies of the string loop oscillations by the parameter of magnetic field B which leads to opposite result than required. In particular the results show that in order to fit all the sources with one model, it is not enough to consider the spherically symmetric black holes with the uniform magnetic field, i.e. a mechanism of decreasing of string loop frequencies is necessary. The role of such a mechanism can play, e.g. the rotation of the black hole, which has been already tested in our previous papers. Other possibility, is to consider more complex configuration of the magnetic field. Preliminary results with the dipole magnetic field configuration shows that the string loop model can explain the observed HF QPOs and allows us to predict the magnetic field intensities in the vicinity of testing sources. More detailed discussion about the oscillations of the string loop near the black hole embedded in an external dipole magnetic field will be given in a future work.

4 CONCLUSIONS

We have calculated the frequencies of the radial and vertical string-loop oscillations in the field of a Schwarzschild black hole immersed in an uniform magnetic field. Unfortunately, it turns out that the effect of the magnetic field is opposite to our expectations and the frequencies obtained by our model in given configuration cannot explain the observed data for all the microquasars GRS 1915+105, XTE 1550-564, GRO 1655-40, see Fig. 2. Clearly, rotation of the black hole is necessary to explain all the observed frequencies in the microquasars by the string loop oscillations (Stuchlík and Kološ, 2014).

ACKNOWLEDGEMENTS

The authors would like to thank the EU grant Synergy CZ.1.07/2.3.00/20.0071, and the internal student grant SGS/23/2013 of the Silesian University. ZS and MK thank the Albert Einstein Centre for gravitation and astrophysics supported by the Czech Science Foundation No. 14-37086G.

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