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Equilibria of charged dust tori in a dipole magnetic field: hydrodynamic approach

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ABSTRACT
We present Newtonian model of non-conductive charged perfect fluid tori orbiting in combined spherical gravitational and dipolar magnetic fields, focusing on stationary, axisymmetric toroidal structures. Matter in such tori exhibits a purely circulatory motion and the resulting convection carries charges into permanent rotation around the symmetry axis. As a main result we demonstrate possible existence of off-equatorial charged tori and equatorial tori with cusps enabling outflows of matter from the torus also in Newtonian regime. From astrophysical point of view, our investigation can provide an insight into processes determining vertical structure of dusty tori surrounding accretion discs.

Keywords: dust tori – electric charge – magnetic field – equilibrium – outflows

1 INTRODUCTION

Supermassive black holes of typical masses $M_\bullet \simeq 10^6$–$10^8\, M_\odot$ are frequently present in nuclei of galaxies, being surrounded by a torus of obscuring material (dust). Moreover, different types of active galactic nuclei in Seyfert galaxies can be unified by introducing some form of obscuring tori, which are believed to encircle the central black hole (Antonucci and Miller, 1985; Urry and Padovani, 1995). The presence of a geometrically and optically thick dusty structure is an essential component of the unification scheme (Hönig and Kishimoto, 2010). The torus structure is thought to be inhomogeneous, in the form of molecular/dusty clumps (clouds).

Equilibrium figures of gaseus tori have been studied in great detail, e.g. in Kozlowski et al. (1978); Abramowicz et al. (1978); Kato et al. (2008), however, the vertical component of the pressure gradient, required to maintain the equilibrium, does not seem to be sufficient in dusty tori (e.g. Murphy and Yaqoob, 2009 and references cited therein). Despite of the fact that signatures of obscuration (especially those seen in X-ray spectra) and variability properties strongly indicate the need for a significant vertical extent of obscuring tori in many Seyfert type 2 galaxies, the physical model for the tori remains uncertain and the vertical structure of dusty tori needs further discussion. For example, it has been proposed by Czerny and Hryniewicz (2011) that vertical motions of the dust clumps play an important role.

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Due to complex electromagnetic processes like photoionisation or plasma electron and ion currents entering the grain surface, the dust particles should possess some net electric charge. In this case, the electrostatic charge is one of the essential parameters that control the dynamics of dust grains embedded in the surrounding cosmic plasma. It was shown, that when electromagnetic forces are taken into account, electrically charged matter can establish vertically extended structures that "levitate" above and under the equatorial plane. The Newtonian study of charged dust grains orbiting in planetary magnetospheres and forming halo orbits were published, e.g. in Howard et al. (1999); Dullin et al. (2002), while the question, if such halo orbits can survive also in strong gravitational fields near compact objects, was successfully answered in Kovář et al. (2008); Stuchlík et al. (2009); Kovář et al. (2010), using both pseudo-Newtonian and general relativistic studies. Of course, in many astrophysical scenarios such simple test-particle approaches fail because of higher densities of charged matter in reality. Then possible approaches follow from the kinetic theory (suitable for lower density matter) or from hydrodynamics (suitable for higher densities).

In this overview we present Newtonian hydrodynamic model of perfect fluid tori with electric charge spread through the fluid of infinite resistivity, which is an opposite limit to the well known ideal magnetohydrodynamics with zero resistivity commonly used to model many astrophysical plasmas (e.g. Punsly, 2001). In more details, the topic presented here is treated in Slaný et al. (2013). General relativistic version of our approach was published in Kovář et al. (2011, 2014) where the charged perfect-fluid tori of infinite resistivity encircling the Reissner–Nordstrøm black hole (without any magnetic field) and Schwarzschild black hole embedded in a homogeneous magnetic field, respectively, were analysed. The kinetic approach suitable for modelling toroidal structures is outlined in Cremaschini et al. (2013).

2 NEWTONIAN MODEL FOR INCOMPRESSIBLE FLUID

The Euler equation for a perfect fluid orbiting in gravitational and electromagnetic fields has the form:

$$\varrho_{\mathrm{m}}\big(\partial_t v_i + v^j \nabla_j v_i\big) = -\nabla_i P - \varrho_{\mathrm{m}} \nabla_i \Phi + \varrho_{\mathrm{e}}\big(E_i + \epsilon_{ijk} v^j B^k\big)\,, \tag{1}$$

where ϱ_{m} and ϱ_{e} are mass-density and charge-density, respectively, P denotes pressure, $\mathbf{v}$ is velocity field in the fluid, and Φ corresponds to the gravitational potential. The electromagnetic field is described by its electric part $\mathbf{E}$ and magnetic part $\mathbf{B}$.

Here, we assume stationary, axisymmetric flow of test charged perfect fluid in external spherical gravitational and dipolar magnetic fields. In spherical polar coordinates $(r,\ \theta,\ \varphi)$

$$\Phi = -\frac{GM}{r}\,, \tag{2}$$

$$E_i = 0\,, \qquad i = (r,\ \theta,\ \varphi)\,, \tag{3}$$

$$B_r = 2\mu \frac{\cos\theta}{r^3}\,, \qquad B_\theta = \mu \frac{\sin\theta}{r^3}\,, \tag{4}$$

where M is the mass of central object and $\mu > 0$ corresponds to magnetic dipole moment of external magnetic field. For stationary, axisymmetric flow

$$v_r = v_\theta = 0\,, \quad v_\varphi = v_\varphi(r,\ \theta)\,. \tag{5}$$

The condition of hydrostatic equilibrium is described by two partial differential equations, following from Euler's equation:

$$\frac{\partial P}{\partial r} = -\varrho_{\mathrm{m}} \frac{GM}{r^2} + \varrho_{\mathrm{m}} \frac{v_\varphi^2}{r} - \varrho_{\mathrm{e}} v_\varphi \mu \frac{\sin\theta}{r^3}\,, \tag{6}$$

$$\frac{1}{r}\frac{\partial P}{\partial \theta} = \varrho_{\mathrm{m}} \frac{v_\varphi^2}{r} \cot\theta + 2\varrho_{\mathrm{e}} v_\varphi \mu \frac{\cos\theta}{r^3}\,. \tag{7}$$

In order to solve this set of equations, it is useful to assume charge density in the form

$$\varrho_{\mathrm{e}} = \varrho_{\mathrm{m}}\, q(r,\,\theta)\,, \tag{8}$$

where $q(r,\,\theta)$ describes specific charge distribution in the fluid. Further we need an equation of state.

The simplest and also very illustrative is the case of incompressible fluid characterized by condition

$$\varrho_{\mathrm{m}} = \mathrm{const}\,. \tag{9}$$

Analysis of integrability conditions for the set of PDEs (6) and (7) reveals that the orbital velocity v_φ could be of the same form as in the uncharged case, i.e.

$$v_\varphi(r,\,\theta) = \mathrm{K}_2 (r\sin\theta)^{\mathrm{K}_1}\,, \tag{10}$$

where K_1 and K_2 are constants which have to be specified. Assuming that the specific charge can be written in a separated form $q(r,\,\theta) = q_1(r)\,q_2(\theta)$, we obtain 4 families of specific charge distribution:

(1) $q(r,\,\theta) = \mathrm{C}\, r^{-3(\mathrm{K}_1-1)/2}$,
(2) $q(r,\,\theta) = \mathrm{C}\, r^{3/2}(\sin\theta)^{-3\mathrm{K}_1}$,
(3) $q(r,\,\theta) = \mathrm{C}\, r^{-3\mathrm{K}_1/2}\sin^3\theta$,
(4) $q(r,\,\theta) = \mathrm{C}\,(\sin\theta)^{3(1-\mathrm{K}_1)}$,

where C is another constant.

In the centre of the torus, the pressure is expected to be maximal, descending monotonically to zero value at the torus surface. Analysis of the condition $\nabla P = 0$ reveals that in the case of charged tori there are two possibilities for torus location: (i) *equatorial torus* with its centre in the equatorial plane ($\theta = \pi/2$), (ii) *off-equatorial torus* with the centre at $\theta \neq \pi/2$.

2.1 Equatorial tori

Uncharged perfect-fluid tori are presented in many classical textbooks on accretion discs, see, e.g. Frank et al. (2002) where also their Newtonian version is presented. These structures are characterized by their equipotential surfaces of "gravito-centrifugal" potential governing the motion of a barotropic fluid in prescribed gravitational field. The equipotential surfaces coincide with isobaric surfaces, P = const. In Newtonian regime, there are closed toroidal surfaces around the circle corresponding to the centre of the torus.

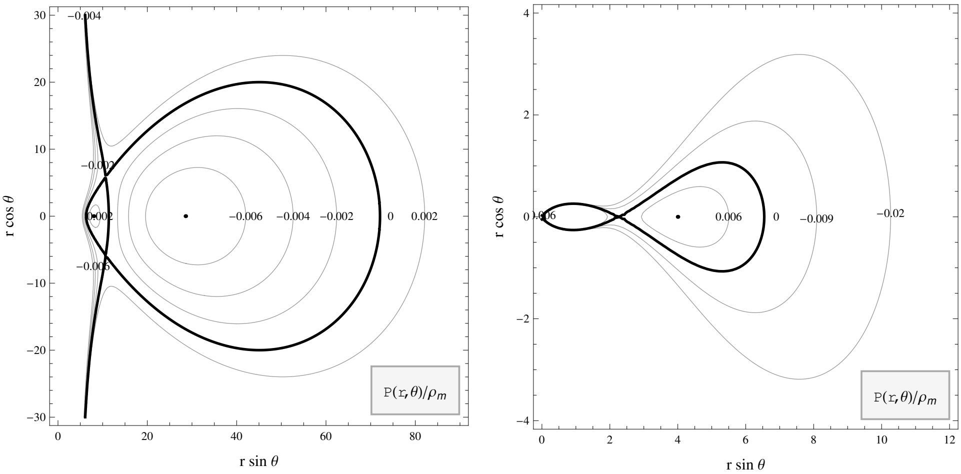

Figure 1. Charged Newtonian tori with cusps. The left panel shows negatively charged torus with cusps out of the equator (physically relevant torus is the small one on the *left*), while the *right* panel shows positively charged torus with the cusp in the equator.

Abramowicz and co-workers showed (Abramowicz et al., 1978; Kozlowski et al., 1978) that in relativistic regime, one of equipotential surfaces can be marginally closed containing the critical point, so-called *cusp*, in the inner edge in the equatorial plane, which enables outflow of matter from the torus and, in fact, accretion onto central compact object. Next, Stuchlík and co-workers showed, see e.g. (Stuchlík et al., 2000; Slaný and Stuchlík, 2005) that cosmic repulsion, represented by the cosmological term in Einstein equations, leads to the existence of another cusp in the structure of equipotential surfaces, now being located at the outer edge in the equatorial plane. For current value of the cosmological constant, however, the outer cusp could be relevant only for very huge toroidal structures of galactic dimensions around supermassive black holes.

In the case of charged tori, we have shown that the cusps can exist also in Newtonian regime and, moreover, that their location is not bound to the equatorial plane only. Structure of isobaric surfaces for cases with cusps is presented in Fig. 1. The left panel describes negatively charged 1^{st}-family torus[1] with uniform distribution of the specific angular momentum $\ell(r,\ \theta) = \mathrm{K}_2 = \mathrm{const}$[2] and spherical distribution of the specific charge. The right panel presents positively charged 4^{th}-family torus with uniform distribution of the specific angular momentum and radial distribution of the specific charge.

2.2 Off-equatorial tori

For negatively charged fluid of the 2^{nd}, 3^{rd} and 4^{th} family-type there exists the possibility of stable off-equatorial tori located symmetrically above and under the equatorial plane. The situation for tori with uniform distribution of the specific angular momentum ($\mathrm{K}_1 = -1$)

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[1] In all presented situations we will expect positive rotation of the torus, i.e. $v_\varphi > 0$ for all fluid elements. In the case of negative rotation, the electric charge of the torus would be opposite.

[2] Tori with constant specific angular momentum correspond to the choice $\mathrm{K}_1 = -1$.

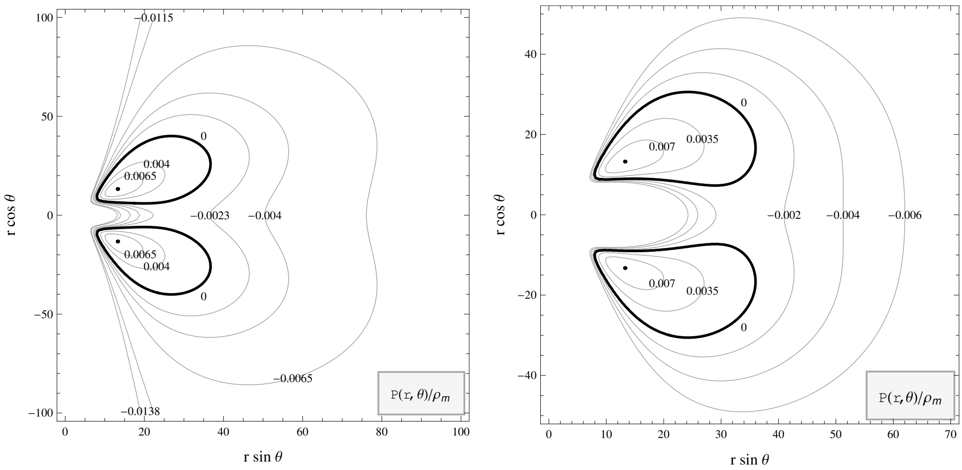

Figure 2. Off-equatorial negatively charged tori with cylindrical (*left panel*) and radial (*right panel*) distribution of the specific charge.

is presented in Fig. 2 where the left panel shows tori with cylindrical distribution of the specific charge ($2^{\rm nd}$-family) while the right panel shows tori with radial distribution of the specific charge ($4^{\rm th}$-family).

3 COMMENTS

For any perfect fluid, the basic set of partial differential equations (PDEs) (6) and (7) has the form

$$\frac{1}{\varrho_{\rm m}}\frac{\partial P}{\partial r} = \mathcal{A}(r,\ \theta)\,, \tag{11}$$

$$\frac{1}{\varrho_{\rm m}}\frac{\partial P}{\partial \theta} = \mathcal{B}(r,\ \theta)\,. \tag{12}$$

If we define a new function

$$h(r,\ \theta) = \frac{P}{\varrho_{\rm m}}\,, \tag{13}$$

the above set of PDEs can be written for incompressible fluid in the form

$$\frac{\partial h}{\partial r} = \mathcal{A}(r,\ \theta)\,, \tag{14}$$

$$\frac{\partial h}{\partial \theta} = \mathcal{B}(r,\ \theta)\,. \tag{15}$$

Now we can think about more general fluid described by polytropic equation of state, $P = K\varrho_{\rm m}^{\gamma}$. Since surfaces $P(r,\theta) = {\rm const}$ coincide with surfaces $\varrho_{\rm m}(r,\theta) = {\rm const}$, due to which they coincide also with surfaces $h(r,\theta) = {\rm const}$, we can use the function h instead of pressure P in the analysis of stationary configurations also for the polytropic fluid. In this case the set (11) and (12) takes the form

$$\frac{\partial h}{\partial r} = \frac{\gamma - 1}{\gamma} \mathcal{A}(r,\, \theta)\,, \tag{16}$$

$$\frac{\partial h}{\partial \theta} = \frac{\gamma - 1}{\gamma} \mathcal{B}(r,\, \theta)\,, \tag{17}$$

being just rescaled version of analogical set (14) and (15) for incompressible fluid. We conclude, therefore, that the results obtained for incompressible fluid are fully relevant also for the polytropic fluid.

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