Black hole spin inferred from disc oscillation models of high-frequency quasi-periodic oscillations

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ABSTRACT
In the past several years, estimations of black hole spin in the three Galactic microquasars GRS 1915+105, GRO J1655-40, and XTE J1550-564 have been carried out based on several models of 3:2 high-frequency quasi-periodic oscillations (HF QPOs). When compared to spin predictions obtained by spectral fitting methods, the different approaches fail to provide consistent results. Most of the so far calculated QPO estimates are implied by models that deal with geodesic accretion flow. In the present work, we assume a non-geodesic flow defined by the model of a pressure-supported perfect fluid torus. We consider several QPO models and explore influence of the consideration of presence of the pressure forces on the predicted QPO frequencies and spin predictions. Our results indicate that in some cases the influence can be quite significant. This is in particular true for the so-called "vertical precession resonance" model and the warped disc resonance model. In other cases, on the other hand, the model predictions do not much vary from those corresponding to geodesic calculations. This applies namely for the model assuming $m = -1$ radial and $m = -2$ vertical disc-oscillation modes. The same is true for the epicyclic resonance (Er) model, but only providing that $a \lesssim 0.9$. When it is $a \gtrsim 0.9$, the situation changes and the influence of pressure forces becomes stronger. Such behaviour leads to very interesting conclusions. Within the Er model framework, individual sources with a moderate spin should exhibit a smaller spread of the measured 3:2 QPO frequencies than sources with a near-extreme spin. This should be further examined using the data available through the proposed Large Observatory for X-ray Timing (LOFT).

1 INTRODUCTION

Studying the X-ray spectra and variability provides a useful tool for putting constraints on the properties of compact objects like is the mass or spin of a black hole. One of the standard ways to measure the spin is through fitting the X-ray spectral continuum or the relativistically broadened Fe K alpha lines (see e.g. McClintock et al., 2006, 2007; Middleton et al., 2006;

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Table 1. Properties of the three microquasars GRO 1655-40, GRS 1915+105, and XTE 1550-564. The individual columns display the frequencies of the lower and upper QPO peaks (Strohmayer, 2001; Remillard et al., 2002, 2003), the mass estimates (Greene et al., 2001; Greiner et al., 2001; Orosz et al., 2002; McClintock and Remillard, 2003), and the spin predictions carried out by the spectral fitting methods.

Source	$\nu_{\rm L}$ [Hz]	$\nu_{\rm U}$ [Hz]	Mass [$M_{\odot}$]	a
GRO 1655-40	300	450	6.0–6.6	0.65–0.80* 0.97–0.99†
GRS 1915+105	113	168	10.0–18.0	0.98–1.00$^{\Delta}$ $\sim 0.7^{\nabla}$
XTE 1550-564	184	276	8.4–10.8	0.75–0.77$^{\ominus}$

*From McClintock et al. (2007). †From Miller et al. (2009). $^{\Delta}$From McClintock et al. (2006).
$^{\nabla}$ From Middleton et al. (2006). $^{\ominus}$From Miller et al. (2009).

Done et al., 2007; Miller, 2007; Shafee et al., 2008; McClintock et al., 2010, 2011, 2014). Within the recent years, another approach has been gaining popularity – the determination of their properties through the theory of high-frequency quasi-periodic oscillations (HF QPOs).

The quasiperiodic modulation of the X-ray flux, which occurs at frequencies comparable to frequencies of orbital motion, has been observed in the X-ray power density spectra of the low-mass X-ray binaries for several decades (see, e.g. van der Klis, 2006; Belloni and Stella, 2014, for a review). In the black hole systems, the HF QPOs appear at frequencies that often form rational ratios with a preferred ratio of 3:2 (Abramowicz and Kluźniak, 2001; McClintock and Remillard, 2003, see Table 1). A significant amount of models proposed to explain the 3:2 HF QPOs deal with orbital motion and some oscillatory modes of the accretion disc. Such models relate the observed QPO frequencies to the corresponding orbital and disc-oscillation frequencies that are often defined by certain combination of the orbital Keplerian frequency and the radial and vertical epicyclic frequencies. In Kerr geometry, these frequencies depend on mass and spin of the black hole, and it is therefore possible to determine the black hole mass or spin from the observed 3:2 QPO frequencies and the specific QPO model. Such spin estimations have been carried out by several authors in the past (Wagoner et al., 2001; Abramowicz and Kluźniak, 2001; Kato, 2004; Török et al., 2005, 2011).

Most of the so-far obtained black hole spin estimations based on the QPO models have been obtained considering a geodesic accretion flow. In the case of more general flows, non-geodesic effects connected to, e.g. pressure gradients, magnetic fields or other forces may have potentially significant impact on the spin predictions implied by these models. Here we aim to quantify such impact in the particular case of non-geodesic influence introduced by pressure forces that are present in a specific type of accretion flow modelled

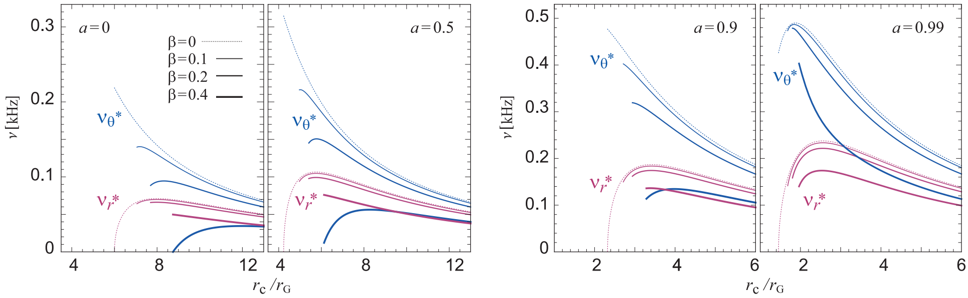

Figure 1. After Šrámková et al. (2015a). Frequencies $\nu_{\rm r}^*$ and ν_θ^* of m = 0 radial and vertical disc-oscillation modes calculated at the centre of the torus, $r = r_{\rm c}$, plotted for various torus thickness (β) and black hole spin a. The calculated frequency values tend to decrease with increasing torus size.

by a pressure-supported, perfect fluid torus. The properties of epicyclic modes of torus oscillations, e.g. modifications to their frequencies due to pressure gradients present in the torus, were calculated by Blaes et al. (2007) in the pseudo-Newtonian approximation and later generalised by Straub and Šrámková (2009) for Kerr geometry.

We assume here several QPO models that were discussed by Török et al. (2011) who calculated spin values predicted by the models dealing with purely geodesic flow for three Galactic microquasars displaying the 3:2 twin-peak HF QPOs – GRS 1915+105, GRO J1655-40, and XTE J1550-564. Using the results of Straub and Šrámková (2009), we carry out the estimates of black hole spin based on the several previously assumed QPO models considering non-geodesic accretion flow of the pressure-supported torus. In this paper, we provide a short summary of the current findings explored by Šrámková et al. (2015a,b).

2 MODEL OF EQUILIBRIUM PRESSURE-SUPPORTED TORUS

The slightly non-geodesic accretion flow considered in this work is modelled by an equilibrium, slightly non-slender pressure-supported perfect fluid torus, which orbits a rotating Kerr black hole and has a constant specific angular momentum distribution. A detailed description of such model of torus is given in Straub and Šrámková (2009). In this accretion flow, the radial and vertical epicyclic oscillations of the fluid are modified by the pressure forces. These modifications were explored by Straub and Šrámková who calculated explicit formulae for the pressure corrections to epicyclic frequencies in a slightly non-slender constant specific angular momentum torus orbiting a Kerr black hole. In Figure 1, we illustrate how the pressure effects modify the frequencies of the axisymmetric m = 0 oscillation modes.

3 DISC-OSCILLATION QPO MODELS

We focus our attention on the so-called 'disc-oscillation' QPO models that involve various oscillatory modes of accretion disc oscillations. The list of the considered models and their corresponding frequency relations of the lower and upper QPO is summarised in

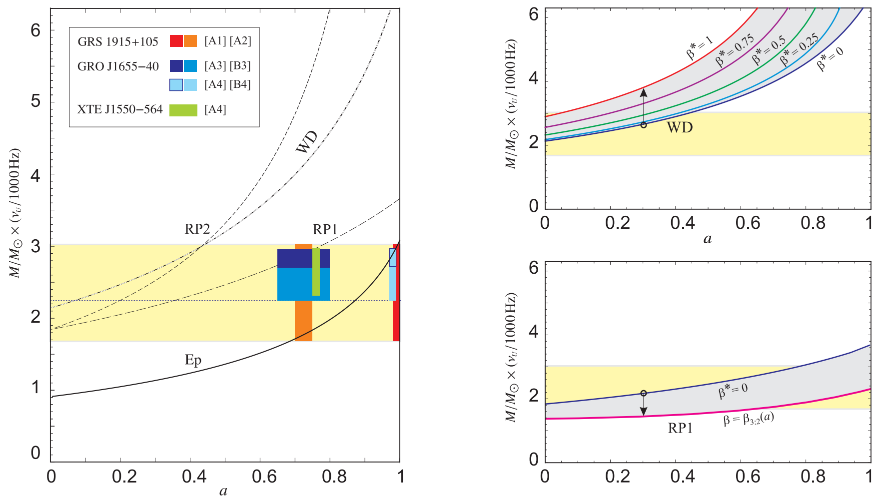

Figure 2. *Left:* After Török et al. (2011). Curves $M(a)$ implied by the individual geodesic models. The light yellow rectangle indicates the observationally determined interval of $\nu_U \times M$ including each of the individual microquasars. The colour boxes are drawn for the QPO independent mass and spin estimates given by different authors. *Right:* Pressure corrections implied for the RP1 and WD model. The geodesic case is marked by the blue line. In the case of WD model, the corrections grow with increasing torus thickness (the value β^* corresponds to the maximal allowed thickness). For the RP1 model, the situation is more complicated. For each value of a, there is a specific limit value of the torus thickness that does not allow the required frequency ratio. The resulting estimates indicated by the shadow area are then carried out numerically.

Table 2. It comprehends the "warped disc" (WD) model (Kato, 2004) that in general assumes oscillation modes in a warped accretion disc. Then there is the "3:2 epicyclic resonance" model of Abramowicz and Kluźniak that attributes the twin-peak HF QPOs to a non-linear resonance between two axisymmetric epicyclic accretion disc oscillation modes. Furthermore, there are another two resonance models that we denote as the "RP1" model (Bursa, 2005) and the "RP2" model (Török et al., 2011). Both of these models deal with a certain combination of non-axisymmetric disc-oscillations modes. More details on these models can be found in Török et al. (2011).

4 SPIN ESTIMATES IMPLIED BY THE NON-GEODESIC QPO MODELS

We use here the formulae for pressure corrections to epicyclic frequencies calculated by Straub and Šrámková. From the 3:2 observed QPO frequencies and estimated ranges of mass of the three microquasars, we infer the spin predicted by the QPO models. For our calculations, we take into account relevant properties of the three microquasars summarized in Table 1. Outputs of these calculations are illustrated in Figs. 2 and 3. The intervals of spin predicted by the individual QPO models carried out for the non-geodesic case are listed in Table 2 and compared to spin predictions calculated by Török et al. (2011).

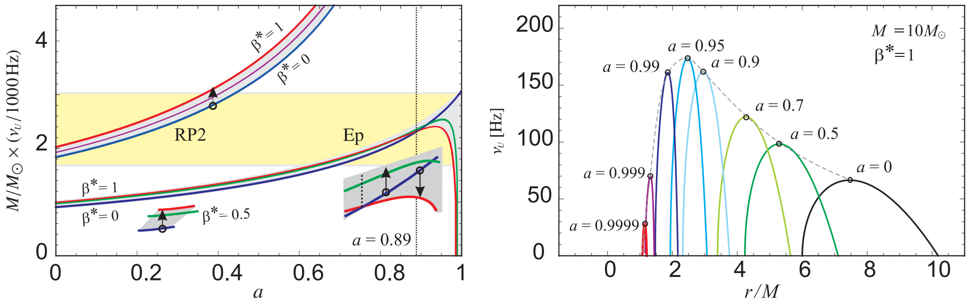

Figure 3. *Left:* Pressure corrections implied for the Ep and the RP2 model. The geodesic case is marked by the blue line. The corrections are rather small except for the case of the Ep model when considering high spin values. *Right:* Frequencies of the m = 0 radial epicyclic mode calculated for tori with cusp tend to increase with increasing spin up to $a \sim 0.95$. For high values of a, they rapidly decrease with increasing a.

5 DISCUSSION AND CONCLUSIONS

Several groups of authors have applied different spectral fitting methods to measure the black hole spin in the three microquasars. We display the intervals of mass and spin of these sources implied by the spectral methods in the mass-spin diagram in the left panel of Fig. 2. The intervals are illustrated in the figure using the several coloured boxes. It is clear from the Figure that the spin predictions carried out by different authors are somewhat inconsistent.

Comparing the spin measurements obtained by the spectral methods to those predicted by theoretical QPO models may help to shed some light on the present puzzling situation. We present such comparison within the mass-spin diagrams displayed in Figs. 2 and 3. The individual curves in the Figures correspond to spin values predicted by the several QPO models given in Table 2. In the left panel of Fig. 2, we show curves corresponding to geodesic-flow estimates calculated by Török et al. (2011), while in the right panel of Fig. 2 and left panel of Fig. 3 we compare these estimates to estimates predicted by QPO models that involve non-geodesic flow described by the equilibrium, pressure-supported fluid torus. Different curves correspond to different torus thickness, which is marked using parameter β^*. Within the adopted notation, the curves marked by $\beta^* = 0$ correspond to the case of a slender torus limit for which the epicyclic frequencies are equal to those of free test particles of geodesic motion. The curves marked by $\beta^* = 1$ then correspond to the case of a torus with cusp.[1]

It is apparent from the Figures that presence of the pressure forces in the accretion flow may imply relatively large modifications to the QPO frequencies and consequently also to spin intervals previously predicted for the geodesic flow. This holds namely for the case of the WD and the RP1 model, both of which are shown in the right panel of Fig. 2.

[1] In the right panel of Fig. 3, we illustrate behaviour of the frequencies of m = 0 radial epicyclic mode calculated for $\beta^* = 1$.

Table 2. Frequency relations corresponding to individual QPO models and the spin of the three microquasars implied by these models for the geodesic (a) and non-geodesic (a^*) case. The relations are expressed in terms of three fundamental frequencies of the perturbed circular geodesic motion. These are the Keplerian frequency, and the radial and vertical epicyclic frequencies, which are denoted by ν_K, ν_r and ν_θ, respectively.

Model	Frequency Relations		$a \sim$	$a^* \sim$
WD	$\nu_\mathrm{L} = 2\,(\nu_\mathrm{K} - \nu_\mathrm{r})$	$\nu_\mathrm{U} = 2\nu_\mathrm{K} - \nu_\mathrm{r}$	<0.45	<0.45
Ep	$\nu_\mathrm{L} = \nu_\mathrm{r}$	$\nu_\mathrm{U} = \nu_\theta$	0.7 – 1	0.6–1
RP1	$\nu_\mathrm{L} = \nu_\mathrm{K} - \nu_\mathrm{r}$	$\nu_\mathrm{U} = \nu_\theta$	<0.80	0–1
RP2	$\nu_\mathrm{L} = \nu_\mathrm{K} - \nu_\mathrm{r}$	$\nu_\mathrm{U} = 2\nu_\mathrm{K} - \nu_\theta$	<0.45	<0.45

The behaviour of curves illustrated in Fig. 3 shows that for the RP2 model assuming $\mathrm{m} = -1$ radial and $\mathrm{m} = -2$ vertical disc-oscillation modes the non-geodesic effects related to pressure do not cause any significant impact. For the Ep model, the results are similar when it is $a \lesssim 0.9$. The situation becomes different for $a \gtrsim 0.9$, in which case the predicted QPO frequency rapidly decreases as the torus thickness rises. This leads to an interesting conclusion for the Ep model. Within the model framework, individual sources with a moderate spin ($a \lesssim 0.9$) should exhibit a smaller spread of the measured 3:2 QPO frequencies than sources with a near-extreme spin, such as GRS-1915+105 ($a \sim 1$). Clearly, this could be further examined using the large amount of high-resolution data available through the proposed Large Observatory for X-ray Timing (LOFT; Feroci et al., 2012).

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