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Braneworld naked singularities

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ABSTRACT
We are investigating singularity structure of the rotating black hole and naked singularity spacetimes in the Randall–Sundrum second type (RSII) of the brane-world scenario. We will show that structure of this singularity is very similar to its classical counterpart, even in the cases of negative tidal charge, which is equivalent to the Kerr–Newman black hole with the complex charge Q (with zero real part). We also study behaviour of the ergosphere and will show that this region can exists under specific situation.

Keywords: Randall Sundrum – Brane-world

1 INTRODUCTION

In recent years, one of the promising approaches to the higher-dimensional gravity theories seems to be the string theory and particularly the M-theory (Hořava and Witten, 1996; Hořava and Witten, 1996). This new idea is describing gravity as a truly higher-dimensional interaction becoming effectively 4D at low enough energies. These theories inspired so called braneworld models, in which the observable universe is a 3-brane on which the standard-model fields are confined, while gravity enters the extra spatial dimensions, the size of which may be much larger than the Planck length scales $\ell_{\mathrm{P}} \sim 10^{-33}$ cm, (Arkani-Hamed et al., 1998). The braneworld models could therefore provide an elegant solution to the hierarchy problem of the electroweak and quantum gravity scales, as these scales become to be of the same order (TeV) due to large scale extra dimensions, (Arkani-Hamed et al., 1998). Therefore, future collider experiments can test the braneworld models quite well, including the hypothetical mini black hole production on the TeV-energy scales, (Dimopoulos and Landsberg, 2001). On the other hand, the braneworld models could be observationally tested since they influence astrophysically important properties of the black holes. Gravity can be localized near the brane at low energies even with a non-compact, infinite size extra dimension with the warped spacetime satisfying the 5D Einstein equations with negative cosmological constant as shown by, (Randall and Sundrum, 1999). In this paper we investigate the influence of the (RSII) brane-world effects on the singularity structure in a Kerr black hole. We also study extension of the ergosphere.

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2 GEOMETRY

Using standard Boyer–Lindquist coordinates (t, r, θ, φ) and geometric units $(c = G = 1)$, we can write the line element of the rotating (Kerr) black hole on the 3D-brane in the form

$$\mathrm{d}s^2 = -\left(1 - \frac{2Mr - b}{\Sigma}\right)\mathrm{d}t^2 - \frac{2a(2Mr-b)}{\Sigma}\sin^2\theta\,\mathrm{d}t\,\mathrm{d}\varphi + \\ + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\,\mathrm{d}\theta^2 + \left(r^2 + a^2 + \frac{2Mr - b}{\Sigma}a^2\sin^2\theta\right)\sin^2\theta\,\mathrm{d}\varphi^2\,, \quad (1)$$

where

$$\Delta = r^2 - 2Mr + a^2 + b\,, \quad (2)$$
$$\Sigma = r^2 + a^2\cos^2\theta\,, \quad (3)$$

M and $a = J/M$ are the mass parameter and the specific angular momentum of the background, while the braneworld parameter b, called “tidal charge”, represents the imprint of non-local (tidal) gravitational effects of the bulk space, (Aliev and Gümrükçüoğlu, 2005).

3 SINGULARITY

Our goal is to find out whether the brane parameter b has strong influence onto the structure of the Kerr-like ring singularity at $r = 0, \theta = \pi/2$. The Kretschmann's scalar $K = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ is a good tool to probe the structure of spacetimes singularities. Using Eq. (1) we get

$$K = \frac{8}{\left(r^2 + a^2t^2\right)^6}\left(r^4A - 2a^2r^2Bt^2 + a^4Ct^4 - 6a^6M^2t^6\right)\,, \quad (4)$$

where[1]

$$t = \cos\theta\,, \quad (5)$$
$$A = \left(7b^2 - 12bMr + 6M^2r^2\right), \quad (6)$$
$$B = \left(17b^2 - 60bMr + 45M^2r^2\right), \quad (7)$$
$$C = \left(7b^2 - 60bMr + 90M^2r^2\right). \quad (8)$$

The Kretschmann scalar is formally same as in the case of the Kerr–Newmann metric with $Q^2 \to b$ (Henry, 2000). Naturally, the negative values of brane parameter would have some effect onto K, but we can see from the denominator of Eq. (4), that it has no effect onto position of the physical singularity. As an example there is a plot of K with $(M = 1, a = 0.8, b = -0.8)$ at Fig. 1.

[1] Substitution $t = \cos\theta$ is used here just to tremendously fasten computation of the Kretschmann scalar by program Mathematica v8.0.

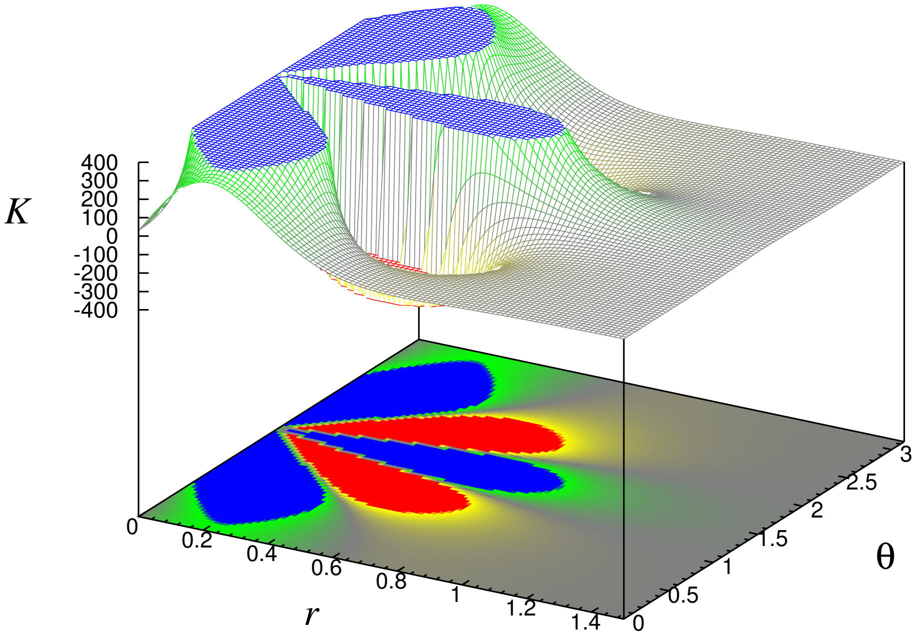

Figure 1. Example of the Kretschmann's scalar K for $M = 1$, $a = 0.8$, $b = -0.8$ to illustrate it's similarity to the Kerr–Newmann case.

Discussion about singularity can be more effectively done if we transform our metric into the so called Kerr–Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} + l_\mu l_\nu \, , \tag{9}$$

where $\eta_{\mu\nu}$ is a flat metric and l_μ is a null vector with respect to $\eta_{\mu\nu}$. Using substitution

$$\mathrm{d}t = \mathrm{d}x^0 + \left(\frac{r^2 + a^2}{\Delta} - 1\right) \mathrm{d}r \, , \tag{10}$$

$$\mathrm{d}\varphi = \mathrm{d}\tilde{\varphi} + \frac{a}{\Delta} \mathrm{d}r \, , \tag{11}$$

$$x = \big(r \cos(\tilde{\varphi}) + a \sin(\tilde{\varphi})\big) \sin\theta \, , \tag{12}$$

$$y = \big(r \sin(\tilde{\varphi}) - a \cos(\tilde{\varphi})\big) \sin\theta \, , \tag{13}$$

$$z = r \cos\theta \, , \tag{14}$$

and after burdensome calculation we end up with metric in a form:

$$\mathrm{d}s^2 = -\big(\mathrm{d}x^0\big)^2 + (\mathrm{d}x)^2 + (\mathrm{d}y)^2 + (\mathrm{d}z)^2 + \frac{(2Mr - b)r^2}{r^4 + a^2 z^2} \times \\ \times \left\{\mathrm{d}x^0 - \frac{1}{r^2 + a^2}\left[r(x\,\mathrm{d}x + y\,\mathrm{d}y) + a(x\,\mathrm{d}y - y\,\mathrm{d}x) - \frac{1}{r} z\,\mathrm{d}z\right]\right\}^2 , \tag{15}$$

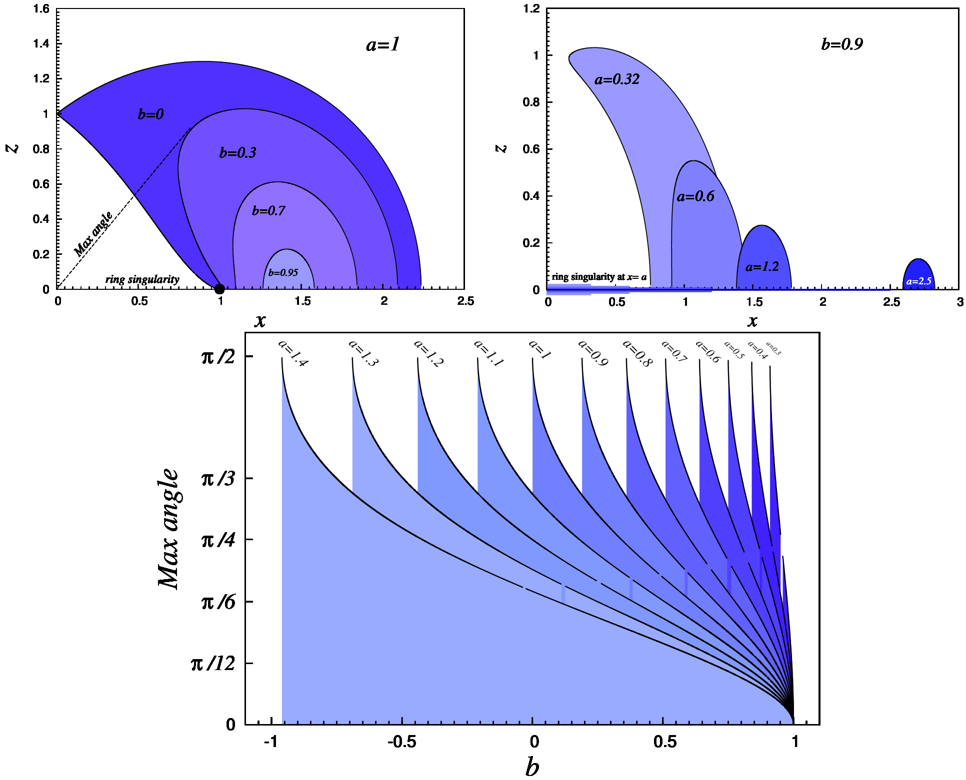

Figure 2. *Upper Left:* Polar slice through the brany Kerr spacetime in Cartesian Kerr–Schild coordinates. Spin parameter a is fixed to value 1 and brany parameter b is appropriately chosen to demonstrate its influence on ergosphere. *Upper Right:* Polar slice through the brany Kerr spacetime in Cartesian Kerr–Schild coordinates. Brany parameter b is fixed to value 0.9 and spin parameter a is appropriately chosen to demonstrate its influence on ergosphere. *Middle:* Maximal possible angle $\alpha = \arctan(z/x)$ for a particular ergosphere.

where r is defined, implicitly, by

$$r^4 - r^2\left(x^2 + y^2 + z^2 - a^2\right) - a^2 z^2 = 0\,. \tag{16}$$

The metric (15) is analytic everywhere except at

$$x^2 + y^2 + z^2 = a^2 \quad \text{and} \quad z = 0\,. \tag{17}$$

This condition is same as in the case of the standard Kerr black hole so we clearly can see that brany parameter b has no influence to singularity of the space-time what so ever. The physical “ring” singularity of the braneworld rotating black holes (and naked singularities) is located at $r = 0$ and $\theta = \pi/2$, as in the Kerr spacetimes. For completeness we also enlist components of Ricci tensor. Ricci scalar is exactly zero, but the braneworld black hole spacetime is not Ricci flat.

Components of Ricci tensor are:

$$R_{tt} = 4b \frac{a^2 + 2\Delta - a^2\cos(2\theta)}{\left(a^2 + 2r^2 + a^2\cos(2\theta)\right)^3}\,, \tag{18}$$

$$R_{t\varphi} = -8ab \frac{\left(a^2 + \Delta\right)\sin^2\theta}{\left(a^2 + 2r^2 + a^2\cos(2\theta)\right)^3}\,, \tag{19}$$

$$R_{\varphi t} = R_{t\varphi}\,, \tag{20}$$

$$R_{rr} = -\frac{R_{\theta\theta}}{\Delta}\,, \tag{21}$$

$$R_{\theta\theta} = \frac{2b}{a^2 + 2r^2 + a^2\cos(2\theta)}\,, \tag{22}$$

$$R_{\varphi\varphi} = 4b\sin^2(\theta) \frac{3a^4 + 2r^4 + a^2\left(b - 2Mr + 5r^2\right) - a^2\Delta\cos(2\theta)}{\left(a^2 + 2r^2 + a^2\cos(2\theta)\right)^3}\,. \tag{23}$$

4 ERGOSPHERE

The ergosphere of Kerr black hole and naked singularities plays a crucial role in astrophysical phenomena related, e.g. to the Penrose process (Penrose and Floyd, 1971), or the ultra high-energy particle collisions. A specially interesting phenomena occur in the case of the naked-singularity spacetimes, (Stuchlík, 1980; Stuchlík and Schee, 2013). Here we explore how the ergosphere extension depends on the tidal charge b and spin a. Ergosphere is a closed area of space with border defined by the condition:

$$g_{tt} = 0\,. \tag{24}$$

It is more convenient to investigate ergosphere in the Kerr–Schild coordinates (15). We can use spacetime symmetry and focus only on polar slice with $y = 0$. In this case the condition for border of ergosphere is simply given by (see for example Visser, 2007)

$$x^2 = \frac{\left(a^2 + r^2\right)\Delta}{a^2}\,,$$
$$z^2 = \frac{(2Mr - b)r^2 - r^4}{a^2}\,. \tag{25}$$

In the Figure 2 we give some examples of the ergosphere's shape. Figures illustrate the influence of the brany parameter b on the ergosphere. From expression for z^2 we see that existence of ergosphere is conditioned by (in $M = 1$ units)

$$b < 1\,. \tag{26}$$

We can also infer that ergosphere is getting larger as brane parameter is getting smaller.

The ergosphere completely surrounds the ring singularity in the black hole cases only. To illustrate this phenomenon we have defined maximal possible angle of ergosphere viewed from the origin of coordinate system:

$$\alpha = \mathrm{Max}: \arctan\frac{z}{x}, \tag{27}$$

where z, x are coordinates of point which belongs to ergosphere (see Fig. 2).

For every positive spin $a > 0$ there always exists an ergospehere, but as spin increases, the volume of the ergosphere and the maximal angle α decreases. The ergosphere is in a sense pushed away from the ring singularity by increasing spin.

5 CONCLUSIONS

We have shown that tidal charge b representing influence of the bulk space on the braneworld has no effect on the effective structure of singularity of the rotating Kerr black hole existing on the brane. Also we have shown how this parameter influences the ergosphere.

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