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HF QPOs in the neutron star binary system XTE J1701-462 fitted by the model of oscillating string loops

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ABSTRACT
Axisymmetric string loops oscillating around a stable equilibrium position in the Kerr background are applied to explain the special set of frequencies related to the high-frequency quasiperiodic oscillations observed in the low mass X-ray binary XTE J1701-462 containing a neutron star. Frequencies of the radial and vertical string loop oscillations are determined by the mass M and dimensionless spin a of the neutron star, and by dimensionless parameter ω describing combined effects of the string loop tension and its angular momentum. Equilibrium position of the string loop is given by its angular momentum and energy. The string-loop oscillation model can explain the observed kHz frequencies, but the stringy parameter ω cannot be the same for all the three HF QPO observations in the XTE J1701-462 source; the limits on the acceptable values of ω are given in dependence on the spacetime parameters M and a. However, the model implies restrict ion $M > 3.3M_{\odot}$ on the neutron star mass that is too high to be compatible with the standard theory of neutron stars. A proper correction on the mass-limit can be generally introduced due to the electromagnetic interaction of an electrically charged string loop with magnetic field of the neutron star.

Keywords: string loops – quasiperiodic oscillations – XTE J1701-462 – X-ray binary

1 INTRODUCTION

The axisymmetric current-carrying string loops are governed by their tension and angular momentum. Tension prevents their expansion beyond some radius, current introduces an angular momentum preventing them from collapse. First, cosmic strings were introduced as remnants of some phase transitions in the very early universe – see (Vilenkin and Shellard, 1995) for a review. Later strings represented as superconducting vortices were introduced by (Witten, 1985). However, the current-carrying string loops could represent also plasma exhibiting a string-like behaviour due to dynamics of the magnetic field lines (Semenov

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et al., 2004; Christensson and Hindmarsh, 1999), or due to the thin flux tubes of magnetized plasma simply described as 1D strings (Semenov and Bernikov, 1991; Cremaschini and Stuchlík, 2013; Cremaschini et al., 2013; Cremaschini and Stuchlík, 2014; Kovář, 2013).

It has been demonstrated that the current-carrying string loops moving axisymmetrically along the symmetry axis of the Kerr or Schwarzschild–de Sitter black holes have significant astrophysical applications (Jacobson and Sotiriou, 2009; Kološ and Stuchlík, 2010; Stuchlík and Kološ, 2012a; Kološ and Stuchlík, 2013; Stuchlík and Kološ, 2014b,a). Transmission of their oscillatory internal energy into energy of the translational motion causes an outward-directed acceleration of the string loops in the strong gravity of stars or compact objects, as neutron stars, black holes, or naked singularities (Jacobson and Sotiriou, 2009; Stuchlík and Kološ, 2012a,b; Kološ and Stuchlík, 2013). Such an effect can be important also for the electrically charged string loops moving in combined gravitational and electromagnetic fields (Tursunov et al., 2013). Since the resulting translational motion can be ultra-relativistic, the transmutation of the string loop energy can serve as an alternative explanation of relativistic jets.

Quite recently, it has been demonstrated that small oscillations of a string loop around stable equilibrium positions in the equatorial plane of the Kerr geometry can be considered in the lowest approximation as two uncoupled linear harmonic oscillators governing the radial and vertical oscillations of the string loop (Kološ and Stuchlík, 2013). The frequencies of the radial and vertical harmonic oscillations of the string loops were given and discussed in (Stuchlík and Kološ, 2014b). It has been shown that the string loop harmonic or quasi-harmonic oscillations can explain frequencies of the twin high-frequency quasiperiodic oscillations (HF QPO) observed in the three Galactic microquasars GRS 1915+105, XTE 1550-564, GRO 1655-40, i.e. low-mass X-ray binary (LMXB) systems containing a black hole (Stuchlík and Kološ, 2014b). Moreover, they can explain also the special frequency set of kHz QPOs observed in the peculiar source XTE J170-407 containing a neutron star, where a single HF QPO and two twin HF QPOs with the frequency ratio 3:2 were observed (Pawar et al., 2013; Stuchlík and Kološ, 2014a). Here we apply the string loop oscillation model for an analogical data set observed in the source XTE J1701-462 containing a neutron star.

The radial profiles of the string loop oscillations qualitatively differ from those related to the radial and vertical oscillations of the geodesic, epicyclic motion of test particle in the Kerr geometry. Especially, there is a crossing point of the radial and vertical frequencies in the Kerr black hole spacetimes for the string loop oscillation allowing for creation of single-frequency peaks to be observed in the field of black holes or neutron stars, while for the test particle oscillations such a crossing is possible only in the Kerr naked singularity spacetimes (Török and Stuchlík, 2005; Stuchlík and Schee, 2012).

2 HF QPOS IN XTE J1701-462

A detailed analysis of the HF QPOs in the source XTE J1701-462 has been reported in (Homan et al., 2007). The results are rather unexpected and very interesting, since a very special set of frequencies has been discovered in this study. In one of the three observational events a single HF QPO has been detected at a characteristic frequency

$$f_{(\mathrm{A})\mathrm{L}} = f_{(\mathrm{A})\mathrm{U}} = 800\,\mathrm{Hz}\,. \tag{1}$$

In the other two observations, twin HF QPOs has been detected at characteristic frequencies

$$f_{(\mathrm{B})\mathrm{L}} = 600\,\mathrm{Hz}\,, \quad f_{(\mathrm{B})\mathrm{U}} = 900\,\mathrm{Hz}\,, \tag{2}$$

$$f_{(\mathrm{C})\mathrm{L}} = 450\,\mathrm{Hz}\,, \quad f_{(\mathrm{C})\mathrm{U}} = 750\,\mathrm{Hz}\,, \tag{3}$$

where we use the index U for the upper and the index L for the lower of the twin frequencies observed simultaneously. The first one of the twin peaks demonstrates precisely the frequency ratio 3:2, while the second one has the frequency ratio 5:3.

We shall use the special character of the radial profiles of the frequencies of the harmonic radial and vertical oscillations of axially symmetric string loops in order to explain the frequency set observed in the source XTE J1701-462 containing a neutron star. We shall assume that the exterior of the neutron star can be well described by the standard Kerr spacetime. Such an assumption is correct for massive neutron stars having mass $M > 2M_{\odot}$ (Urbanec et al., 2013).

3 DYNAMICS OF STRING LOOPS

Dynamics of an axisymmetric current-carrying string loop in a given axially symmetric and stationary, Kerr, spacetime with metric $g_{\alpha\beta}$ has been studied in detail in (Jacobson and Sotiriou, 2009; Kološ and Stuchlík, 2013). Harmonic or quasiharmonic oscillations of string loops in the Kerr spacetimes have been studied in (Stuchlík and Kološ, 2014b). Here we give a short overview.

The string loop motion is governed by barriers due to the tension and the angular momentum that are modified by the gravitational field. Dynamics of the string loop is determined by the action

$$S = \int \mathrm{d}^2\sigma \sqrt{-h}\big(\mu + h^{ab}\varphi_{,a}\varphi_{,b}\big)\,, \tag{4}$$

where $\varphi_{,a} = j_a$ determines current of the string loop, $\mu > 0$ reflects the string tension, and h^{ab} represents the metric induced on the string worldsheet. The worldsheet stress-energy tensor density $\tilde{\Sigma}^{ab}$ can be expressed in the form (Jacobson and Sotiriou, 2009; Kološ and Stuchlík, 2013),

$$\tilde{\Sigma}^{\tau\tau} = \frac{J^2}{g_{\phi\phi}} + \mu\,, \quad \tilde{\Sigma}^{\sigma\sigma} = \frac{J^2}{g_{\phi\phi}} - \mu\,, \quad \tilde{\Sigma}^{\sigma\tau} = \frac{-2j_\tau j_\sigma}{g_{\phi\phi}}\,, \quad J^2 \equiv j_\sigma^2 + j_\tau^2\,. \tag{5}$$

The parameters $J^2 = j_\tau^2 + j_\sigma^2$ and $\omega = -j_\tau/j_\sigma$ describe the angular momentum of the string loop (Kološ and Stuchlík, 2010; Stuchlík and Kološ, 2012b)

As demonstrated in (Larsen, 1993), the string loop motion can be described by the Hamilton equations related to the 4-momentum P_μ with the Hamiltonian

$$H = \frac{1}{2}g^{rr}\,P_r^2 + \frac{1}{2}g^{\theta\theta}\,P_\theta^2 + \frac{1}{2}g_{\phi\phi}\big(\Sigma^{\tau\tau}\big)^2 + \frac{g_{\phi\phi}\big(E + g_{t\phi}\Sigma^{\sigma\tau}\big)^2}{2\big(g_{tt}g_{\phi\phi} - g_{t\phi}^2\big)}\,. \tag{6}$$

The Hamiltonian can be written as a sum of dynamic and potential parts

$$H = H_{\mathrm{D}} + H_{\mathrm{P}} = \frac{1}{2} g^{rr} P_r^2 + \frac{1}{2} g^{\theta\theta} P_\theta^2 + H_{\mathrm{P}}(r, \theta) \,. \tag{7}$$

Using the conserved energy $E = -p_t$ and the angular momentum parameters J and ω (5), the potential part of the Hamiltonian reads

$$H_{\mathrm{P}} = \frac{1}{2} g_{\phi\phi} \left(\frac{J^2}{g_{\phi\phi}} + 1 \right)^2 + \frac{1}{2} \frac{g_{\phi\phi}}{g_{tt} g_{\phi\phi} - g_{t\phi}^2} \left(E + \frac{g_{t\phi}}{g_{\phi\phi}} \frac{2J^2\omega}{\omega^2 + 1} \right)^2 \,. \tag{8}$$

The boundary of the string loop motion is given by the condition $H_{\mathrm{P}} = 0$ that implies the energy boundary function $E_b(r, \theta)$ in the form (Kološ and Stuchlík, 2013; Stuchlík and Kološ, 2014b),

$$E = E_{\mathrm{b}}(r, \theta) = \sqrt{g_{t\phi}^2 - g_{tt} g_{\phi\phi}} \, \tilde{\Sigma}^{\tau\tau} - g_{t\phi} \tilde{\Sigma}^{\sigma\tau} \,. \tag{9}$$

The energy boundary function $E_{\mathrm{b}}(r, \theta)$ governs the dynamics of the string loops, serving as an effective potential of their motion. The rescaling $E/\mu \to E$ and $J/\sqrt{\mu} \to J$ implies that the energy boundary function in the standard Boyer–Lindquist r, θ coordinates (Carter, 1973) takes the form

$$E_{\mathrm{b}}(r, \theta; a, J, \omega) = \frac{4a\omega J^2 r}{\left(\omega^2 + 1\right) G} + \sqrt{\Delta} \left(\frac{J^2 R^2}{G \sin(\theta)} + \sin(\theta) \right) , \tag{10}$$

where

$$G(r, \theta; a) = \left(a^2 + r^2\right) R^2 + 2a^2 r \sin^2(\theta) \,. \tag{11}$$

In the Kerr metric

$$R^2 = r^2 + a^2 \cos^2\theta \,, \quad \Delta = r^2 - 2Mr + a^2 \,, \tag{12}$$

where a denotes spin and M mass parameters of the Kerr spacetimes. Of course, for the exterior of neutron stars we have to consider only the part of the Kerr spacetime limited by the condition $r \geq R_{\mathrm{surface}} > r_+$.

In the following, we shall use for simplicity the dimensionless radial coordinate $r \to r/M$, dimensionless time coordinate $t \to t/M$ and dimensionless spin $a \to a/M$; this is equivalent to using of $M = 1$ in the metric tensor. We will return to the dimensional quantities in the Section 5.

Detailed discussion of the properties of the energy boundary function $E_{\mathrm{b}}(r, \theta)$ is presented in (Kološ and Stuchlík, 2013) for both the Kerr black hole and naked singularity spacetimes. Here we focus on the properties in the black hole spacetimes that can be relevant for rotating neutron stars as demonstrated in (Urbanec et al., 2013; Török et al., 2008) – in this case the local extrema of the energy boundary function can be located in the equatorial plane only.

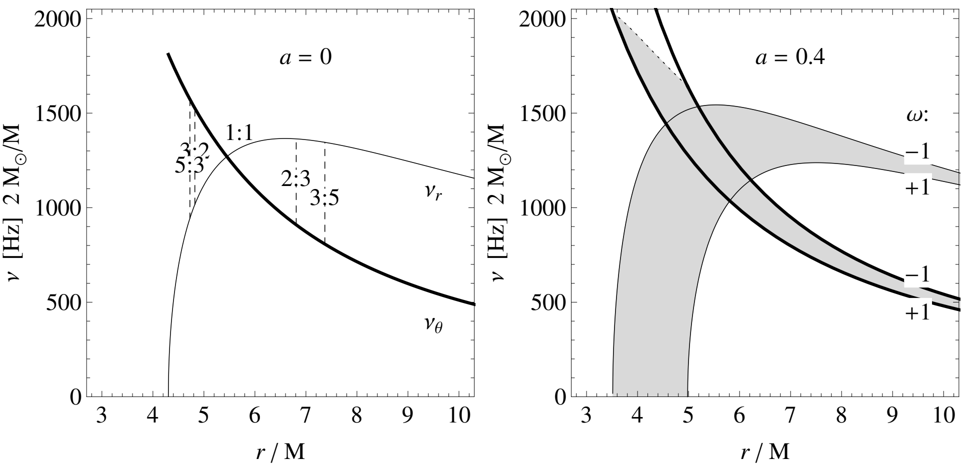

Figure 1. String-loop oscillatory frequencies $\nu_{\rm r}$ (*thin curves*) and ν_θ (*thick curves*), calculated for the Kerr metrics with $M = 2M_\odot$. Their radial profiles are illustrated for values of dimensionless spin $a = 0, 0.4$ that are characteristic of our study of neutron star system. We demonstrate extension of the frequency radial profiles for the complete range of the string loop parameter $\omega \in \langle -1, 1\rangle$. The vertical frequency curves are restricted to the region of existence (zero point) of the corresponding radial frequency curves – the relevant region is greyed.

The local extrema of the energy boundary function $E_{\rm b}(r; a, J, \omega)$, governing the equilibrium positions of the string loops in the equatorial plane ($\theta = \pi/2$), are determined by the function $J^2_{\rm E}(r; a, \omega)$ defined by (Kološ and Stuchlík, 2013; Stuchlík and Kološ, 2014b)

$$J^2_{\rm E}(r; a, \omega) = \frac{(r-1)\left(\omega^2+1\right) H^2}{4a\omega\sqrt{\Delta}\left(a^2+3r^2\right) + \left(\omega^2+1\right) F}\,, \tag{13}$$

where

$$H(r; a) = r^3 + a^2(2+r)\,, \quad F(r; a) = (r-3)r^4 - 2a^4 + a^2 r\left(r^2 - 3r + 6\right). \tag{14}$$

A detailed discussion of the properties of the energy boundary function $E_{\rm b}(r; a, J, \omega)$ and the string loop motion can be found in (Kološ and Stuchlík, 2013; Stuchlík and Kološ, 2014b). We have to concentrate on the situations when for a string loop with fixed values of the angular momentum parameters J and ω a stable equilibrium position of the string loop exists being given by the equation

$$J^2 = J^2_{\rm E}(r; a, \omega)\,. \tag{15}$$

Around such stable equilibrium positions, small oscillations of string loops occur, if their energy slightly exceeds the minimal value of the effective potential at the stable equilibrium positions.

4 RADIAL AND VERTICAL OSCILLATIONS OF CURRENT-CARRYING STRING LOOPS AND THEIR FREQUENCIES

The analysis of the oscillatory motion of string loops around their stable equilibrium positions, based on the perturbative treatment of the Hamiltonian, can be found in (Kološ and Stuchlík, 2013; Stuchlík and Kološ, 2014b). The generally chaotic motion of the string loops can be in the first approximation of the motion around the stable equilibrium position considered as a regular motion – the corresponding part of the perturbative Hamiltonian represents two uncoupled linear harmonic oscillatory modes for the motion in the radial and vertical directions.

For string loop harmonic oscillations around a stable equilibrium position the variations of the radial and latitudinal coordinates are governed by the equations

$$\ddot{\delta r} + \omega_{\mathrm{r}}^2\,\delta r = 0\,, \quad \ddot{\delta\theta} + \omega_{\theta}^2\,\delta\theta = 0\,. \tag{16}$$

The locally measured angular frequencies are given by (Stuchlík and Kološ, 2014b) and read

$$\omega_{\mathrm{r}}^2 = \frac{1}{g_{rr}}\frac{\partial^2 H_{\mathrm{P}}}{\partial r^2}\,, \quad \omega_{\theta}^2 = \frac{1}{g_{\theta\theta}}\frac{\partial^2 H_{\mathrm{P}}}{\partial \theta^2}\,, \tag{17}$$

where the partial derivatives of the potential part of the Hamiltonian are calculated at the local minimum of the energy boundary function. The locally measured angular frequencies are connected to the angular frequencies related to distant observers, $\Omega_{(\mathrm{r},\theta)}$, by the gravitational redshift transformation (Stuchlík and Kološ, 2014b),

$$\Omega_{(\mathrm{r},\theta)} = \frac{\mathrm{d}f_{(\mathrm{r},\theta)}}{\mathrm{d}t} = \frac{\omega_{(\mathrm{r},\theta)}}{P^t}\,. \tag{18}$$

If the angular frequencies $\Omega_{(\mathrm{r},\theta)}$, or frequencies $\nu_{(\mathrm{r},\theta)}$, of the string loop oscillation are expressed in the physical units, their dimensionless form has to be extended by the factor c^3/GM. Then the frequencies of the string loop oscillations measured by the distant observers are given by

$$\nu_{(\mathrm{r},\theta)} = \frac{1}{2\pi}\frac{c^3}{GM}\,\Omega_{(r,\theta)}\,. \tag{19}$$

This is the same factor as the one occurring in the case of the orbital and epicyclic frequencies of the geodesic motion in the Kerr spacetime (Aliev and Galtsov, 1981; Török and Stuchlík, 2005; Stuchlík and Schee, 2012). The order of magnitude and the mass-scaling of the frequencies of the radial and vertical oscillations is the same for both the current-carrying string loops and test particles, therefore the string loop oscillations could serve as an alternate explanation of the HF QPOs observed in the strong gravity regions of black holes and neutron stars.

The angular frequencies of the string loop oscillations related to distant observers take the dimensionless form

$$\Omega_{\mathrm{r}}^2(r;a,\omega) = \frac{J_{\mathrm{E(ex)}}\left(2a\omega\sqrt{\Delta}\left(a^2+3r^2\right)+\left(\omega^2+1\right)F_1\right)}{2r\left(a^2(r+2)+r^3\right)^2 F_3^2}\,, \tag{20}$$

$$\Omega_\theta^2(r;a,\omega) = \frac{\sqrt{\Delta}\left(2a\omega\sqrt{\Delta}\left(2a^2-3a^2r-3r^3\right)+\left(\omega^2+1\right)F_2\right)}{r^2\left(a^2(r+2)+r^3\right)F_3}\,, \tag{21}$$

where

$$F_1(r,a) = a^2r^3 - a^2\Delta + r^5 - 2r^4\,, \tag{22}$$

$$F_2(r;a) = a^4(3r-2) + 2a^2(2r-3)r^2 + r^5\,, \tag{23}$$

$$F_3(r;a,\omega) = 2a\omega\left(a^2+3r^2\right) + \sqrt{\Delta}\left(\omega^2+1\right)\left(r^3-a^2\right), \tag{24}$$

$$\begin{aligned} J_{\mathrm{E(ex)}}(r;a,\omega) = {} & \left(\omega^2+1\right)H(r-1)\left(6a^2r-3a^2r^2-6a^2-5r^4+12r^3\right) \\ & + 4a\omega H\Delta^{-1/2}\left[\left(a^2+3r^2\right)\left(\Delta-(r-1)^2\right)-6\Delta r(r-1)\right] \\ & - \left(\omega^2+1\right)\left[FH+2F\left(a^2+3r^2\right)(r-1)\right] \\ & + 8a\omega\sqrt{\Delta}\left(a^2+3r^2\right)^2(r-1)\,. \end{aligned} \tag{25}$$

The function $J_{\mathrm{E(ex)}}(r;a,\omega)$ governs the local extrema of the function $J_{\mathrm{E}}(r;a,\omega)$. Its zero points determine the marginally stable equilibrium positions of the string loops. The conditions

$$J_{\mathrm{E(ex)}} = 0 \quad \text{and} \quad J_{\mathrm{E}}^2 \geq 0\,, \tag{26}$$

satisfied simultaneously, put the limit on validity of the formulae giving the angular frequencies of the radial and vertical oscillations – for details see (Stuchlík and Kološ, 2014b).

The radial profiles of the frequencies of the radial and vertical string loop harmonic oscillations are demonstrated in Fig. 1 for two characteristic values of the Kerr spin parameter $a = 0, 0.4$. In the Schwarzschild spacetime ($a = 0$), both the frequencies are independent of the parameter ω. In the Kerr spacetimes, the range of the radial and vertical frequencies depends on the string-loop parameter ω, and the spin parameter a of the spacetime. Extension of the range of allowed frequencies increases with increasing spin. For all values of the spin and at each radius where the two oscillatory modes can occur, the vertical frequency has its maximum (minimum) for string loops with $\omega = -1$ ($\omega = +1$), while the radial frequency has its maximum (minimum) for string loops with $\omega = +1$ ($\omega = -1$); see Fig. 1.

5 STRING-LOOP OSCILLATIONS AS A MODEL OF HF QPOS IN THE XTE J1701-462 SOURCE

The rotating neutron stars can be conveniently described by the Hartle–Thorne geometry. Recently it has been demonstrated that agreement of the external Hartle–Thorne and Kerr geometries is sufficiently high for neutron stars with the mass $M > 2M_\odot$, the dimensionless spin $a < 0.5$, and the relative quadrupole moment $q/a^2 < 2$ (Urbanec et al., 2013). Here we assume that the XTE J1701-462 neutron star is at a state enabling description of its exterior

Table 1. Four possible combinations of HF QPOs observed in the XTE J1701-462 source.

		$f_{(A)}$	$f_{(B)}$	$f_{(C)}$
case 1	$\nu_r : \nu_\theta$	1:1	3:2	5:3
case 2	$\nu_r : \nu_\theta$	1:1	3:2	3:5
case 3	$\nu_r : \nu_\theta$	1:1	2:3	5:3
case 4	$\nu_r : \nu_\theta$	1:1	2:3	3:5

by the Kerr geometry – we shall see that the predictions of the string loop oscillation model put high limits on the mass and dimensionless spin of the neutron star, in agreement with this assumption.

The upper theoretical constraint on the neutron star mass based on realistic equations of state of the subnuclear matter reads

$$M < M_{\mathrm{maxNS}} = 2.8\,\mathrm{M}_{\odot}\,, \quad a < 0.5 \tag{27}$$

(see e.g. Akmal and Pandharipande, 1997; Akmal et al., 1998; Chamel et al., 2013). The upper theoretical limit on the neutron star spin reads $a_{\mathrm{maxNS}} \sim 0.7$, (Lo and Lin, 2011).

For a given twin HF QPOs observed in a given source, we have to consider fixed values of the string parameter ω and the spacetime parameters M and a. If several twin HF QPOs are observed in the source, the spin and mass parameters have to be fixed, but the string loop parameter ω can be varied, as different twin frequency observations could be generated by different string loops that could be created and decayed successively with different parameter ω reflecting locally different conditions in the source. Therefore, the string-loop oscillation model naturally introduces a possibility of significant scatter in distribution of frequencies of the twin HF QPOs. The range of the scatter increases with increasing spin of the Kerr geometry.

Here we assume relevance of resonant phenomena, e.g. a parametric resonance (Landau and Lifshitz, 1969), at all of the three HF QPO events observed in the XTE J1701-462 source. We consider the rational frequency ratios $\nu_\theta : \nu_r = 3{:}2$ or $\nu_\theta : \nu_r = 5{:}3$ for the twin HF QPOs, and $\nu_\theta : \nu_r = 1:1$ for the single HF QPO, to be directly related to the observed values of the QPO frequencies in the XTE J1701-462 source. We identify the frequencies ν_U, ν_L with ν_θ, ν_r or ν_r, ν_θ frequencies. There are four possible combinations of this identification, enabled by the properties of the string loop oscillation model. The resonant phenomena between the radial and vertical oscillatory modes can occur at resonant radii $r_{1:1}$, $r_{3:2}$, $r_{2:3}$, $r_{5:3}$, $r_{3:5}$. The four possible cases of their combination are presented in Table 2.

The fitting of the string loop oscillation frequencies to the observed frequencies is presented in Fig. 2. for all the four cases of possible combinations of the resonant radii of the string loop oscillations. At each of the three observed events, and each of the resonant radii, the fitting is related to the upper of the observed frequencies (or the common frequency at $r_{1:1}$); the precision of the frequency measurement is also taken into account. The fitting procedure determines for each of the observed events a region of the $M - a$ parameter space, determined by the limiting values of the string loop parameter $\omega \in \langle -1, 1\rangle$. Due to the degeneracy of the radial profiles of the string loop oscillation frequencies in the Schwarzschild

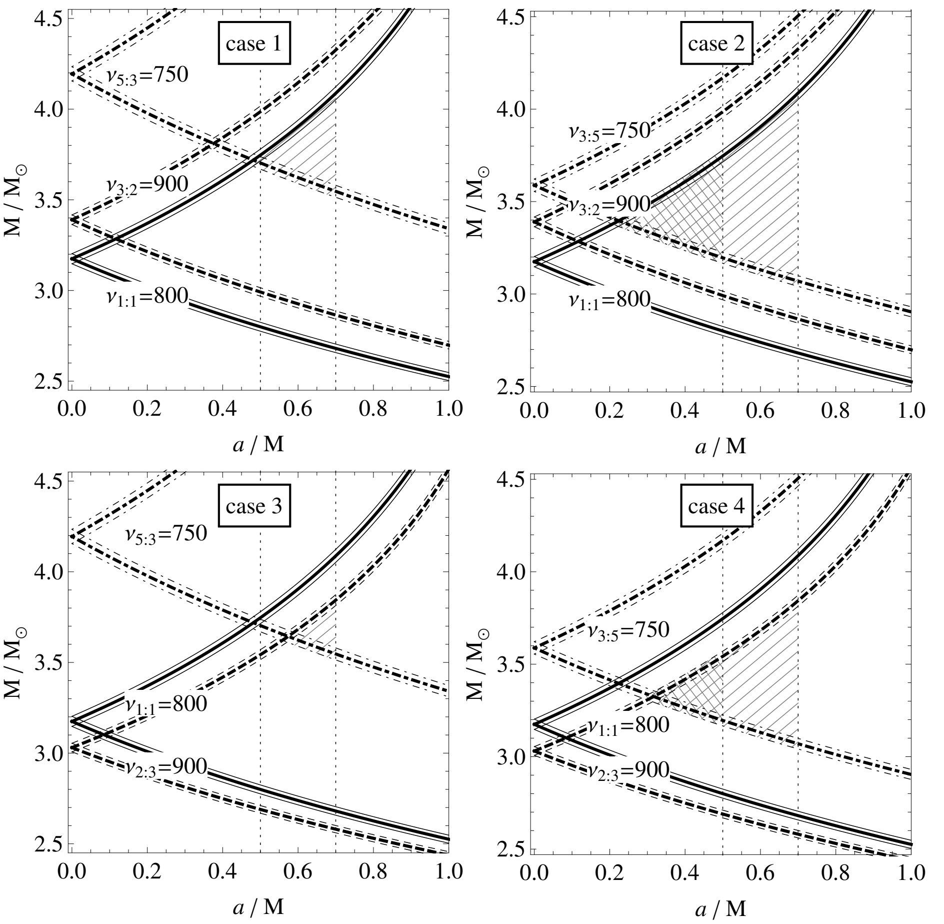

Figure 2. Restrictions on the mass M and spin a parameters of the neutron star in the XTE J1701-462 source implied by the string loop oscillation model applied to the three observational events of HF QPOs at the source. We assume that the three observational events occur at the resonant points of the radial and vertical string loop oscillations. Four different cases of the combinations of the resonant points related to the three observational events are possible – see Table 1. The upper branches are for parameter $\omega = +1$ and the lower branches for parameter $\omega = -1$, allowed regions of the spacetime parameters M, a are hatched. The most promising is the second case, where we consider the 1:1 resonance with frequency $\nu_{1:1} = 800$ Hz combined with 3:5 and 3:2 resonances with frequencies $\nu_{3:5} = 750$ Hz and $\nu_{3:2} = 900$ Hz.

spacetimes ($a = 0$), the fitting predicts only one value of the mass parameter M for the spin $a = 0$ at each observational event. Extension of the allowed region related to the whole interval of string loop parameter $\omega \in \langle -1, 1\rangle$ (the interval of allowed values of M) increases with increasing spin a. The string loop oscillation model thus implies a "triangular" limit on the spacetime parameters M, a for each of the observed events of HF QPOs, as shown in Fig. 2. The limits have to be satisfied simultaneously, and we thus directly obtain the

Table 2. Restriction on the parameters of the neutron star in the XTE J1701-462 source implied by the string loop oscillation model of HF QPOs. Presented values correspond to the hatched regions from Fig. 2.

case 1:

	$a = 0.48$	$a = 0.5$	$a = 0.7$
$M/\mathrm{M}_{\odot}$	3.72	3.70 – 3.75	3.55 – 4.09
$\omega_{1:1}$	1	0.69 – 1	0.27 – 1
$\omega_{3:2}$	0.33	0.29 – 0.34	0.09 – 0.44
$\omega_{5:3}$	-1	-1 – -0.61	-1 – -0.08

case 2:

	$a = 0.22$	$a = 0.5$	$a = 0.7$
$M/\mathrm{M}_{\odot}$	3.40	3.20 – 3.75	3.07 – 4.09
$\omega_{1:1}$	1	0.03 – 1	-0.08 – 1
$\omega_{3:2}$	0.01	-0.24 – 0.34	-0.31 – 0.44
$\omega_{3:5}$	-1	-1 – 0.19	-1 – 0.39

case 3:

	$a = 0.57$	$a = 0.5$	$a = 0.7$
$M/\mathrm{M}_{\odot}$	3.64	$\nexists$	3.55 – 3.84
$\omega_{1:1}$	0.44	$\nexists$	0.27 – 0.49
$\omega_{2:3}$	1	$\nexists$	0.45 – 1
$\omega_{5:3}$	-1	$\nexists$	-1 – -0.27

case 4:

	$a = 0.32$	$a = 0.5$	$a = 0.7$
$M/\mathrm{M}_{\odot}$	3.32	3.20 – 3.54	3.07 – 3.84
$\omega_{1:1}$	0.26	0.03 – 0.40	-0.08 – 0.49
$\omega_{2:3}$	1	0.22 – 1	0.06 – 1
$\omega_{3:5}$	-1	-1 – -0.03	-1 – 0.22

region of allowed values of the spacetime parameters, if the theoretical restrictions on the spin ($a < 0.5$ related to the Hartle–Thorne model of neutron stars) are also taken into account. (We demonstrate in Fig. 2. also the limit related to the fully general-relativistic restriction on the neutron star spin, $a < 0.7$). Along with the restrictions on the spacetime parameters M and a, restrictions on the stringy parameter ω are obtained simultaneously. The results representing the limits on the spacetime and string loop parameters M, ω are presented in Table 2 for the characteristic limiting values of the spin parameter a. Of course, the restrictions on the radii where the resonant oscillations occur have to be also taken into the account, if we test the Hartle–Thorne models of the neutron star for concrete equations of state.

Our results indicate that only the cases 2 and 4, with both the resonant radii related to $r_{3:5}$, are physically realistic, as the other two cases, where the radius $r_{5:3}$ enters the play, give unrealistic values of the neutron star spin. Using the results obtained in the case 2, we can see that for the lowest value of the spin, $a = 0.2$, we obtain the unique value of the neutron star mass, $M = 3.4\,M_{\odot}$. In the edge of the allowed range of spin ($a = 0.5$), we can obtain the mass $M = 3.2\,M_{\odot}$. This is above the range of neutron star mass applied by realistic equations of state, being on the theoretical limit given by general restrictions on the neutron star mass as discussed in (Ruffini, 1973). Note that a similar situation, with one single and two twin HF QPOs, occurs for the neutron star source XTE J1701-407, but in this case the restrictions implied by the string loop oscillation model on the spin and mass of the neutron star are in accord with realistic equations of state, as shown in (Stuchlík and Kološ, 2014a). The resonant radii are depicted along with the radial profiles of the radial and vertical frequencies in Fig. 3 for some characteristic allowed values of the spacetime parameters M and a.

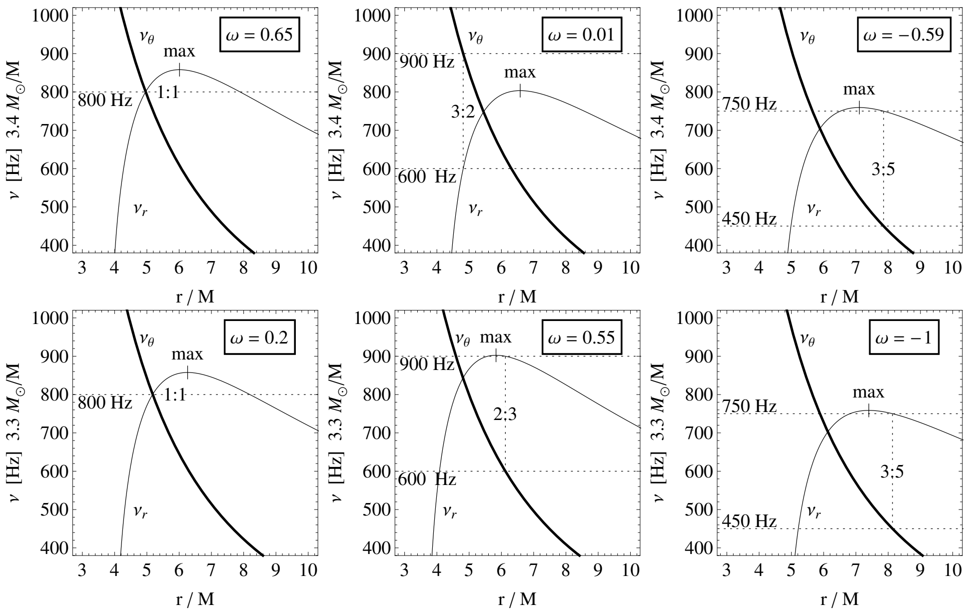

Figure 3. Examples of the radial profiles of the string-loop oscillatory frequencies ν_{r} (*thin curves*) and ν_{θ} (*thick curves*) as related to the three observational events are given for two representative situations allowed by the combination of the resonant points in the case 2 and 4. The parameters of the Kerr metric are mass $M = 3.4\,M_{\odot}$ and spin $a = 0.25$ for the first row (case 2), $M = 3.3\,M_{\odot}$ and spin $a = 0.35$ for the second row (case 4). The related values of the parameter ω are depicted in all the subfigures. Relevant resonant frequencies are also given.

6 CONCLUSIONS

We have demonstrated that the three HF QPOs observed in the XTE J1701-462 LMXB source containing a neutron star can be formally explained by the string loop oscillating model introduced in (Stuchlík and Kološ, 2014b) for the oscillations in the Kerr spacetime. This model, reflecting oscillations of string loops governed by interplay of tension and angular momentum, gives relevant restrictions on the spacetime parameters M, a and the string loop parameter ω that must be varied for the three observational events. We cannot fit the observed data assuming only one string loop having a fixed value of the parameter ω reflecting locally different conditions in the source.

The string-loop oscillation model implies that the neutron star spacetime parameters are restricted to the intervals $0.2 < a < 0.4$ and $3.3 < M/M_{\odot} < 3.6$ predicting thus a very massive and fast rotating neutron star. Since the neutron star has to be very massive, we can conclude that the application for the Kerr geometry in the fitting procedure could be justified, as for the near-maximum-mass neutron stars the exterior Hartle–Thorne geometry has to be close to the exterior Kerr geometry, giving close predictions of the physical phenomena occurring in their vicinity. However, the predicted mass is too high to be acceptable for realistic equations of state. Therefore, our results indicate that if the string loop oscillation model has to be relevant for the HF QPOs observed in the XTE J1701-462 source, an electrically charged string loop interacting with the neutron star magnetic field has to be invoked in order to allow for mass acceptable due to the Hartle-Thorne model using realistic equations of state (Tursunov et al., 2014; Stuchlík and Kološ, 2014a).

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REFERENCES

Akmal, A. and Pandharipande, V. R. (1997), Spin-isospin structure and pion condensation in nucleon matter, *Phys. Rev. C*, **56**, pp. 2261–2279, arXiv: `nucl-th/9705013`.

Akmal, A., Pandharipande, V. R. and Ravenhall, D. G. (1998), The equation of state for nucleon matter and neutron star structure, *ArXiv Nuclear Theory e-prints*, arXiv: `nucl-th/9804027`.

Aliev, A. N. and Galtsov, D. V. (1981), Radiation from relativistic particles in nongeodesic motion in a strong gravitational field, *General Relativity and Gravitation*, **13**, pp. 899–912.

Carter, B. (1973), Black hole equilibrium states., in C. Dewitt and B. S. Dewitt, editors, *Black Holes (Les Astres Occlus)*, pp. 57–214.

Chamel, N., Haensel, P., Zdunik, J. L. and Fantina, A. F. (2013), On the Maximum Mass of Neutron Stars, *Internat. J. Modern Phys. A*, **22**, 1330018, arXiv: `1307.3995`.

Christensson, M. and Hindmarsh, M. (1999), Magnetic fields in the early universe in the string approach to MHD, *Phys. Rev. D* , **60**(6), 063001, arXiv: `astro-ph/9904358`.

Cremaschini, C. and Stuchlík, Z. (2013), Magnetic loop generation by collisionless gravitationally bound plasmas in axisymmetric tori, *Phys. Rev. E* , **87**(4), 043113.

Cremaschini, C. and Stuchlík, Z. (2014), Transition from gas to plasma kinetic equilibria in gravitating axisymmetric structures, *Phys. Plasmas*, **21**(4), 042902.

Cremaschini, C., Stuchlík, Z. and Tessarotto, M. (2013), Kinetic theory of quasi-stationary collisionless axisymmetric plasmas in the presence of strong rotation phenomena, *Phys. Plasmas*, **20**(5), p. 052905.

Homan, J. et al. (2007), Rossi X-Ray Timing Explorer Observations of the First Transient Z Source XTE J1701-462: Shedding New Light on Mass Accretion in Luminous Neutron Star X-Ray Binaries, *Astrophys. J.*, **656**, pp. 420–430, arXiv: `astro-ph/0610803`.

Jacobson, T. and Sotiriou, T. P. (2009), String dynamics and ejection along the axis of a spinning black hole, *Phys. Rev. D* , **79**(6), 065029, arXiv: `0812.3996`.

Kološ, M. and Stuchlík, Z. (2010), Current-carrying string loops in black-hole spacetimes with a repulsive cosmological constant, *Phys. Rev. D* , **82**(12), 125012, arXiv: `1103.4005`.

Kološ, M. and Stuchlík, Z. (2013), Dynamics of current-carrying string loops in the Kerr naked-singularity and black-hole spacetimes, *Phys. Rev. D* , **88**(6), 065004, arXiv: `1309.7357`.

Kovář, J. (2013), Spiral motion formation in astrophysics, *European Physical Journal Plus*, **128**, p. 142.

Landau, L. D. and Lifshitz, E. M. (1969), *Mechanics*, Oxford: Pergamon Press.

Larsen, A. L. (1993), Dynamics of cosmic strings and springs; a covariant formulation, *Classical and Quantum Gravity*, **10**, pp. 1541–1548, arXiv: `hep-th/9304086`.

Lo, K.-W. and Lin, L.-M. (2011), The Spin Parameter of Uniformly Rotating Compact Stars, *Astrophys. J.*, **728**, 12, arXiv: 1011.3563.

Pawar, D. D., Kalamkar, M., Altamirano, D., Linares, M., Shanthi, K., Strohmayer, T., Bhattacharya, D. and van der Klis, M. (2013), Discovery of twin kHz quasi-periodic oscillations in the low-mass X-ray binary XTE J1701-407, *Monthly Notices Roy. Astronom. Soc.*, **433**, pp. 2436–2444, arXiv: 1306.0168.

Ruffini, R. (1973), On the energetics of black holes., in C. Dewitt and B. S. Dewitt, editors, *Black Holes (Les Astres Occlus)*, pp. 451–546.

Semenov, V., Dyadechkin, S. and Punsly, B. (2004), Simulations of Jets Driven by Black Hole Rotation, *Science*, **305**, pp. 978–980, arXiv: astro-ph/0408371.

Semenov, V. S. and Bernikov, L. V. (1991), Magnetic flux tubes - Nonlinear strings in relativistic magnetohydrodynamics, *Astrophys. and Space Sci.*, **184**, pp. 157–166.

Stuchlík, Z. and Kološ, M. (2012a), Acceleration of string loops in the Schwarzschild-de Sitter geometry, *Phys. Rev. D* , **85**(6), 065022, arXiv: 1206.5658.

Stuchlík, Z. and Kološ, M. (2012b), String loops in the field of braneworld spherically symmetric black holes and naked singularities, *Journal of Cosmology and Astroparticle Physics* , **10**, 008, arXiv: 1309.6879.

Stuchlík, Z. and Kološ, M. (2014a), String loop oscillation model fitting frequency of kHz quasiperiodic oscillations in the low-mass X-ray binary XTE J1701-407, *submitted to Classical and Quantum Gravity*.

Stuchlík, Z. and Kološ, M. (2014b), String loops oscillating in the field of Kerr black holes as a possible explanation of twin high-frequency quasiperiodic oscillations observed in microquasars, *Phys. Rev. D* , **89**(6), 065007, arXiv: 1403.2748.

Stuchlík, Z. and Schee, J. (2012), Observational phenomena related to primordial Kerr superspinars, *Classical and Quantum Gravity*, **29**(6), 065002.

Török, G., Bakala, P., Stuchlík, Z. and Čech, P. (2008), Modeling the Twin Peak QPO Distribution in the Atoll Source 4U 1636-53, *Acta Astronomica* , **58**, pp. 1–14.

Török, G. and Stuchlík, Z. (2005), Radial and vertical epicyclic frequencies of Keplerian motion in the field of Kerr naked singularities. Comparison with the black hole case and possible instability of naked-singularity accretion discs, *Astronomy and Astrophysics*, **437**, pp. 775–788, arXiv: astro-ph/0502127.

Tursunov, A., Kološ, M., Ahmedov, B. and Stuchlík, Z. (2013), Dynamics of an electric current-carrying string loop near a Schwarzschild black hole embedded in an external magnetic field, *Phys. Rev. D* , **87**(12), 125003.

Tursunov, A., Kološ, M., Ahmedov, B. and Stuchlík, Z. (2014), Acceleration of electric current-carrying string loop near a Schwarzschild black hole immersed in an asymptotically uniform magnetic field, *submitted into Phys. Rev. D* .

Urbanec, M., Miller, J. C. and Stuchlík, Z. (2013), Quadrupole moments of rotating neutron stars and strange stars, *Monthly Notices Roy. Astronom. Soc.*, **433**, pp. 1903–1909, arXiv: 1301.5925.

Vilenkin, A. and Shellard, E. P. S. (1995), *Cosmic Strings and Other Topological Defects*, Cambridge University Press.

Witten, E. (1985), Superconducting strings, *Nuclear Physics B*, **249**, pp. 557–592.