Proceedings of *RAGtime 14–16, 18–22 Sept./15–18 July/11–19 Oct., '12/'13/'14, Opava, Prague, Czech Republic* 257
Z. Stuchlík, G. Török and T. Pecháček, editors, Silesian University in Opava, 2014, pp. 257–267

Determination of Characteristics of Eclipsing Binaries with Spots: Phenomenological vs Physical Models

Mariia G. Tkachenko[a] and Ivan L. Andronov[b]

Odessa National Maritime University, Mechnikov st. 23, UA-65029 Odessa, Ukraine
[a]`masha.vodn@yandex.ua`
[b]`tt_ari@ukr.net`

ABSTRACT
We discuss methods for modelling eclipsing binary stars using the "physical", "simplified", and "phenomenological" models. There are few realizations of the "physical" Wilson–Devinney (1971) code and its improvements, e.g. Binary Maker, Phoebe. A parameter search using the Monte-Carlo method was realized by Zola et al. (2010), which is efficient in expense of too many evaluations of the test function. We compare existing algorithms of minimization of multi-parametric functions and propose to use a "combined" algorithm, depending on if the Hessian matrix is positively determined. To study methods, a simply fast-computed function resembling the "complete" test function for the physical model. Also we adopt a simplified model of an eclipsing binary at a circular orbit assuming spherical components with an uniform brightness distribution. This model resembles more advanced models in a sense of correlated parameter estimates due to a similar topology of the test function. Such a model may be applied to detached Algol-type systems, where the tidal distortion of components is negligible.

Keywords: variable stars – eclipsing binaries – algols – data analysis – time series analysis – parameter determination

1 INTRODUCTION

Determination of the model parameters of various astrophysical objects, comparison with observations and, if needed, further improvement of the model, is one of the main directions of science, particularly, of the study of variable stars. And so we try to find the best method for the determination of the parameters of eclipsing binary stars. For this purpose, we have used observations of one eclipsing binary system, which was analysed by (Zola et al., 2010). This star is AM Leonis, which was observed using 3 filters (B, V, R). For the analysis, we used the computer code written by Professor Stanisław Zoła (Zola et al., 1997). In the program, the Monte-Carlo method is implemented. As a result, the parameters were determined and the corresponding light curves are presented in the paper (Andronov and Tkachenko, 2013)

978-80-7510-127-3

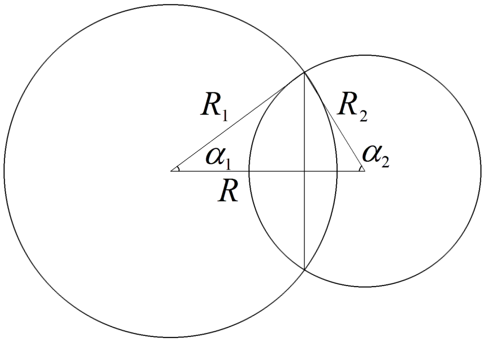

Figure 1. Scheme of eclipsing binary system with spherical components. Number 1 corresponds to a larger star, which eclipses the smaller star at phases close to 0.

With an increasing number of evaluations, the points are being concentrated to smaller and smaller regions. And, finally, the "cloud" should converge to a single point. Practically this process is very slow. This is why we try to find more effective algorithms. At the "potential – potential" diagram (Andronov and Tkachenko, 2013), we see that the best solution corresponds to an "over-contact" system, which makes an addition link of equal potentials $\Omega_1 = \Omega_2$ and corresponding decrease of the number of unknown parameters.

Such a method needs a lot of computation time. We had made fitting using a hundred thousands sets of model parameters. The best 1500 (user defined) points are stored in the file and one may plot the "parameter – parameter diagrams". Of course, the number of parameters is large, so one may choose many pairs of parameters. However, some parameters are suggested to be fixed, and thus a smaller number of parameters is to be determined.

Looking for the "parameter–parameter" diagrams, we see that there are strong correlations between the parameters, e.g. the temperature in our computations is fixed for one star. If not, the temperature difference is only slightly dependent on temperature, thus both temperatures may not be determined accurately from modelling. So the best solution may not be unique; it may fill some sub-space in the space of parameters.

This is a common problem: the parameter estimates are dependent. Our tests were made on another function, which is similar in behaviour to a test function used for modelling of eclipsing binaries.

To determine the statistically best sets of the parameters, there are some methods for optimization of the test function which is dependent on these parameters (Cherepashchuk, 1993; Kallrath and Milone, 2009). As for the majority of binary stars the observations are not sufficient to determine all parameters, for smoothing the light curves may be used "phenomenological fits". Often were used trigonometric polynomials (="restricted Fourier series"), following a pioneer work of (Pickering, 1881) and other authors, see (Parenago and Kukarkin, 1936) for a detailed historical review. Andronov (2010, 2012) proposed a method of phenomenological modelling of eclipsing variables (most effective for algols, but also applicable for EB and EW – type stars).

Below we discuss the "simplified" and "phenomenological" models.

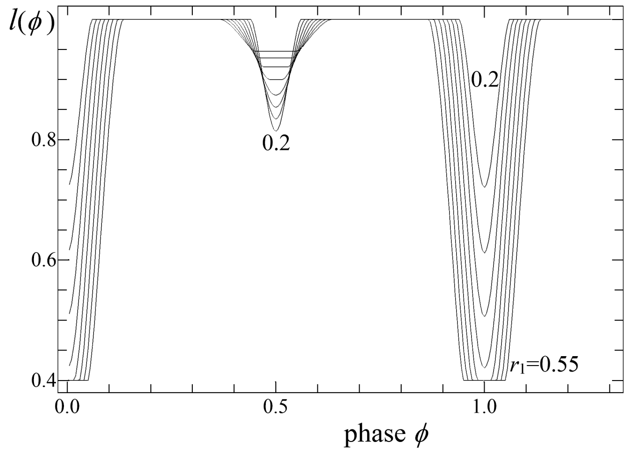

Figure 2. A set of theoretical light curves for the "simplified" model generated for R_1 in a range from 0.2 to 0.55 with a step of 0.05 for fixed values of other parameters listed in the text.

2 "SIMPLIFIED" MODEL

The simplest model is based on the following main assumptions: the stars are spherically symmetric (this is physically reliable for detached stars with components being deeply inside their Roche lobes); the surface brightness distribution is uniform. This challenges the limb darkening law, but is often used for teaching students because of simplicity of the mathematical expressions, e.g. (Andronov, 1991). Similar simplified model of an eclipsing binary star is also presented by Dan Bruton (`http://www.physics.sfasu.edu/astro/ebstar/ebstar.html`). The scheme is shown in Fig. 1. The parameters are L_1, L_2 (proportional to luminosities), radii R_1, R_2, distance R between the projections of centres to the celestial sphere.

The square of the eclipsed segment is $S = S_1 + S_2$

$$S_1 = R_1^2(\alpha_1 - \sin\alpha_1 \cos\alpha_1)\,, \tag{1}$$

$$S_2 = R_2^2(\alpha_2 - \sin\alpha_2 \cos\alpha_2)\,, \tag{2}$$

where the angles a_1, a_2 may be determined from the cosine theorem:

$$\cos\alpha_1 = \frac{R^2 + R_1^2 - R_2^2}{2R_1R} = \frac{R^2 + \eta}{2R_1R}\,, \tag{3}$$

$$\cos\alpha_2 = \frac{R^2 + R_2^2 - R_1^2}{2R_1R} = \frac{R^2 - \eta}{2R_2R}\,, \tag{4}$$

where obviously $\eta = R_1^2 - R_2^2$ and $|R_1 - R_2| \leq R \leq |R_1 + R_2|$. The total flux is $L = L_1 + L_2$, if $R \geq R_1 + R_2$ (i.e. both stars are visible, $S = 0$). For $R \leq R_1 + R_2$, $S = \pi R_2^2$ (assuming that $R_2 \leq R_1$). Generally, $L = L_1 + L_2 - L_j S/\pi R_j^2$, where j is the number of star which is behind another, i.e. $j = 1$, if $\cos 2\pi\phi \leq 0$, and $j = 2$, if $\cos 2\pi\phi \geq 0$. Here ϕ is phase ($\phi = 0$) corresponds to a full eclipse, independently on which star has larger brightness). For scaling purposes, a dimensionless variable $l(\phi) = L(\phi)/(L_1 + L_2)$ is usually introduced. For tests, we used a light curve generated for the following parameters: $R_1 = 0.3$, $R_2 = 0.2$, $L_1 = 0.4$, $L_2 = 0.6$ and $i = 80°$. The phases were computed with a step of 0.02. This light curve as well as other generated for a set of values of R_1 is shown in Fig. 2. As a test function we have used:

$$F = \sum_{i=1}^{n} \frac{\left(x_i - \alpha x_c(\phi_i)\right)^2}{\sigma_i^2}, \tag{5}$$

where x_i (or l_i) are values of the signal at phases ϕ_i with a corresponding accuracy estimate σ_i, and x_c are theoretical values computed for a given trial set of m parameters. For normally distributed errors and absence of systematic differences between the observations and theoretical values, the parameter F is a random variable with χ^2_{n-m} a statistical distribution (Anderson, 1984; Cherepashchuk, 1993). For the analysis carried out in this work, we used a simplified model with $\sigma_i = 1$. This assumption does not challenge the basic properties of the test function. The scaling parameter is sometimes determined as $x(0.75)/x_c(0.75)$, i.e. at a phase where both components are visible, and the flux (intensity) has its theoretical maximum (in the "no spots" model). To improve statistical accuracy, it may be recommended to use a scaling parameter computed for all real observations:

$$\alpha = \frac{\sum_{i=1}^{n} \frac{x_i}{\sigma_i^2}}{\sum_{i=1}^{n} \frac{x_c(\phi_i)}{\sigma_i^2}}, \tag{6}$$

This corresponds to a least squares estimate of a scaling parameter, i.e. the model value of the out–of–eclipse intensity $L = L_1 + L_2$ may be theoretically an any positive number, and these parameters may be "independent". By introducing $l_1 = L_1/L$ and $l_2 = L_2/L$, we get an obvious relation $l_2 = 1 - l_1$, i.e. one parameter. For L sometimes are used values at the observed light curve at the phase 0.75 (i.e. when both stars are to be visible so maximal light). We prefer instead to use all the data with scaling as in Eq. (6). Even in our simplified model, the number of parameters is still large (4). At Figure 4, the lines of equal levels of F are shown. One may see that the zones of small values are elongated and inclined showing a high correlation between estimates of 2 parameters. In fact this correlation is present for other pairs of parameters. This means that there may be relatively large regions in the multi-parameter space which produce theoretical light curves of nearly equal coincidence with observations.

In the software by Zola et al. (2010), the Monte-Carlo method is used, and at each trial computation of the light curve, the random parameters C_k, $k = 1, 2, 3, 4$ are used in a corresponding range: $C_k = C_{k,\min} + (C_{k,\max} - C_{k,\min})$ rand, where rand is an uniformly distributed random value. Then one may plot "parameter – parameter" diagrams for "best" points after a number of N trial computations. The "best" means sorting of sets of the parameters according to the values of the test function F. Initially, the points are distributed

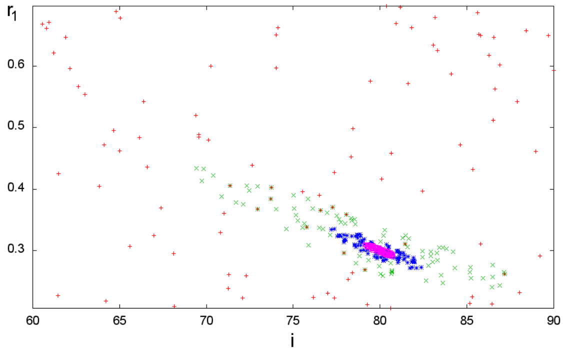

Figure 3. The "parameter–parameter" diagram. The best (according to the value of the test function) 100 points after 10^2, 10^3, 10^4, 10^5 trial computations, respectively.

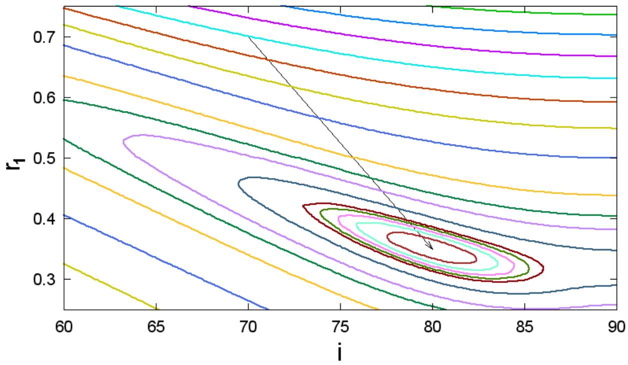

Figure 4. Lines of equal values of the test function F for fixed values of other parameters. The arrow shows position of the "true" parameters used to generate the signal.

uniformly. With an increasing N, "better" (with smaller F) point concentrate to a minimum. There may be some local minima, if the number of parameters will be larger (e.g. spot(s) present in the atmosphere(s) of component(s)). We had made computations for an artificial function of $m(= 1, 2, 3)$ variables (Andronov and Tkachenko, 2013). The minimal value δ (as a true value was set to zero), which was obtained using N trial computations in the Monte-Carlo method is statistically proportional to

$$\delta \propto N^{-2/m}, \tag{7}$$

i.e. the number of computations $N \propto \delta^{-m/2}$ drastically increases with both an increasing accuracy and number of parameters. For our simplified model, the numerical experiments statistically support this relation. Also, the distance between the "successful computations" (when the test function becomes smaller than all previous ones) $\Delta N \propto N$. Obviously, it is not realistic to make computations of the test function for billions times to get a set of statistically optimal parameters. In the "brute force" method, the test functions are computed using a grid in the m – dimensional space, so the interval of each parameter is divided by n_i points. The number of computations is $N = n_1 n_2 \ldots n_m$ should be still large. Either the Monte Carlo method, or the "brute force" one allow to determine positions of the possible local extrema in an addition to the global one. However, if the preliminary position is determined, one should use faster methods to reach the minimum. Classically, there may be used the method of the "steepest descent" (also called the "gradient descent"), where the new set of parameters may be determined as

$$C_{k+1,i} = C_{k,i} - \lambda h_{k,i} \,, \tag{8}$$

where $C_{k,i}$ is the estimated value of the coefficient C_i at k-th iteration, $h_{k,i}$ – proposed vector of direction for the coefficient C_i, and λ is a parameter. Typically one may use one of the methods for one-dimensional minimization (Press et al., 2007; Korn and Korn, 1968), determine a next set of the parameters $C_{k,i}$, recompute a new vector $h_{k,i}$ and again minimize λ. In the method of the steepest descent, one may use a $h_{k,i} = \partial F/\partial C_i$ gradient as a simplest approximation to this vector. Another approach (Newton–Raphson) is to redefine a function $F(\lambda) = F(C_i, i = 1 \ldots m)$, compute the root of equation $\partial F/\partial\lambda = 0$, and then to use a parabolic approximation to this function. Thus

$$\lambda = \Big(\partial F/\partial\lambda)/(\partial^2 F/\partial\lambda^2\Big) \,. \tag{9}$$

There may be some modifications of the method based on a decrease of λ, which may be recommended, if the shape of the function significantly differs from a parabola. In the method of "conjugated gradients", the function is approximated by a second-order polynomial. Finally it is usually recommended to use the (Marquardt, 1963) algorithm. We tested this algorithm with a combination of the "steepest descent" (when the determinant of the Hessian matrix is negative) and "conjugated gradients" (if positive), which both are efficient for a complex behaviour of the test function.

3 PHENOMENOLOGICAL MODELLING

Besides physical modelling of binary stars, there are methods, which could be introduced as "phenomenological" ones. In other words, we apply approximations with some phenomenological parameters, which have no direct relation to physical parameters – masses, luminosities, radii etc. The most often used are algebraic polynomial approximations, included in the majority of computer programs (e.g. electronic tables like Microsoft Office Excel, Libre/Open Office Calc, GNUmeric etc.). For periodic processes, one can use a trigonometric polynomial (also called "restricted sum of Fourier series")

$$\begin{aligned} x_c(\phi, s) &= C_1 + \sum_{j=1}^{s}\big(C_{2j}\cos(2j\pi\phi) + C_{2j+1}\sin(2j\pi\phi)\big) \\ &= C_1 + \sum_{j=1}^{s} R_j \cos\big(2j\pi(\phi - \phi_j)\big) \,. \end{aligned} \tag{10}$$

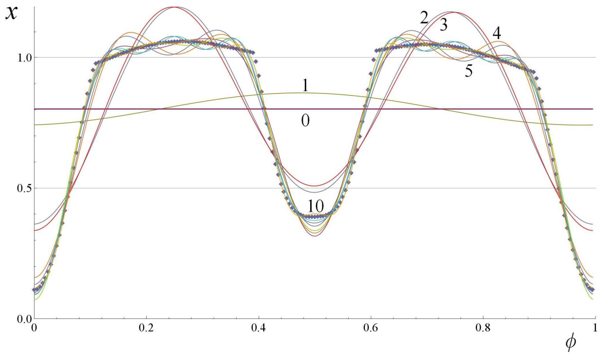

Figure 5. Trigonometrical polynomial approximations of the phenomenological light curve. The degree s is shown by numbers near corresponding curves.

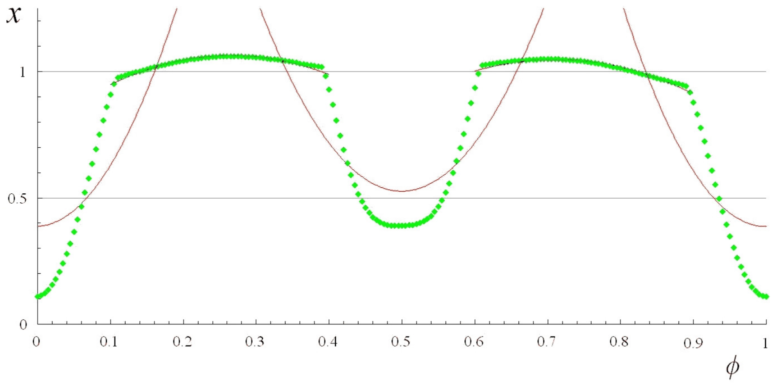

Figure 6. The model light curve and its approximation by parabola at the intervals of phases centered on mainima and maxima, as proposed by (Papageorgiou et al., 2014)

The upper Equation is used for determination (using the Least Squares method) of the parameters C_α, $\alpha = 1 \ldots m$, where the number of parameters is $m = 1 + 2s$, where the lower converts the pairs of the coefficients C_{2j+1}, C_{2j+1} for each $(j-1)$-th harmonic according to usual relations

$$\begin{aligned}
&C_{2j} = R_j \cos(2j\pi\phi_0)\,, \\
&C_{2j+1} = R_j \sin(2j\pi\phi_0)\,, \\
&R_j = \left(C_{2j}^2 + C_{2j+1}^2\right)^{1/2}\,, \\
&\phi_j = \mathrm{atan}(C_{2j+1}/C_{2j})/2\pi + 0.25\big(1 - \mathrm{sign}(C_{2j})\big)\,.
\end{aligned} \tag{11}$$

Here $j = 1 \ldots j_{\max}$, $j_{\max} = n/2$ for even n and $j_{\max} = (n-1)/2$ for odd n. Using the Least Squares algorithm, it is possible to determine parameters C_α even for irregularly spaced data e.g. (Andronov, 1994). *Only* under strong conditions $\phi_k = \phi_0 + k/n$, where

$k = 0 \ldots n-1$, n is the number of observations, one may obtain simplified expressions for the "Discrete Fourier Transform" (DFT) as an extension of the original Fourier (1822) method:

$$C_0 = \frac{1}{n}\sum_{k=0}^{n-1} x_k\,,$$
$$C_{2j} = \frac{2}{n}\sum_{k=0}^{n-1} x_k \cos(2j\pi k/n)\,, \tag{12}$$
$$C_{2j+1} = \frac{2}{n}\sum_{k=0}^{n-1} x_k \sin(2j\pi k/n)\,.$$

If $j = n/2$, then

$$C_n = \frac{1}{n}\sum_{k=0}^{n-1} (-1)^k x_k\,, \tag{13}$$
$$C_{n+1} = 0\,.$$

For irregularly spaced data, there are at least 6 different modifications of the method, which are called themselves as "Fourier Transform", and give same correct results only under assumptions listed above for the DFT. For irregularly spaced data The links may be found in (Andronov, 2003).

Theoretically, the degree of the trigonometric polynomial s is infinite for continuous case (number of data $n \to \infty$) and should be $s = j_{\max} = \mathrm{int}(n/2)$, i.e. may be a large number. For this case, one will get an interpolating function. For lower degree $s < j_{\max}$, the function is smoothing, and one may use different criteria for choosing the statistically optimal value, e.g. the Fischer's criterion (or equivalent one based on the Beta-type distribution), the criterion of minimum of r.m.s. error estimate of the smoothing function (at the moments of observations; integrated over all interval; or at some specific value of the argument), or the maximum of the "signal–to–noise" ratio.

However, these sums may show apparent waves (so called Gibbs phenomenon). It may be illustrated in Fig. (5) for a sample function. One may see different approximations. With an increasing s, the approximation $x_c(\phi, s)$ becomes closer (in a sense of the Least Squares), but the apparent waves are well pronounced at $m \ll n$.

To decrease the number of parameters, (Andronov, 2010, 2012) proposed an approximation combined from a second-degree trigonometric polynomial and a local function modelling the shape of the eclipses:

$$\begin{aligned} x_c(\phi) = & C_1 + C_2\cos(2\pi\phi) + C_3\sin(2\pi\phi) + \\ & + C_4\cos(4\pi\phi) + C_5\sin(4\pi\phi) + \\ & + C_6 H(\phi-\phi_0, C_8, \beta_1) + C_7 H(\phi-\phi_0+0.5, C_8, \beta_2)\,. \end{aligned} \tag{14}$$

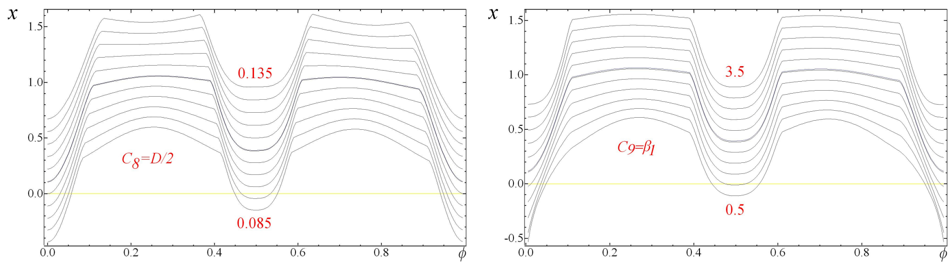

Figure 7. Dependencies of the light curves (intensity vs. phase) on the parameters $C_8 = D/2$ (*left*) and $C_9 = \beta_1$ (*right*). The relative shift in intensity between subsequent curves is 0.1. The *thick line* shows a best fit curve

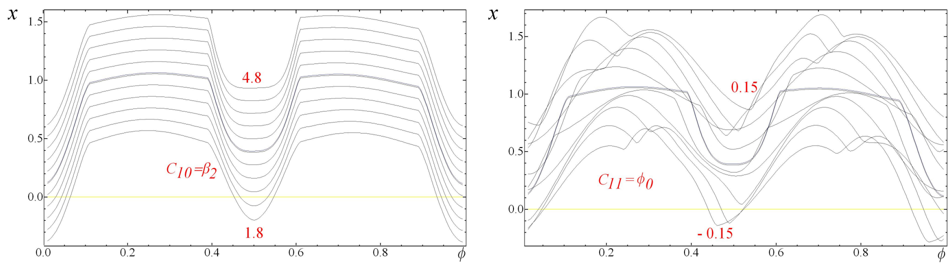

Figure 8. Dependencies of the light curves on the parameters $C_{10} = \beta_2$ (left) and $C_{11} = \phi_0$ (right).

$$H(\phi, C_8, \beta) = \begin{cases} V(z) = \left(1 - |z|^\beta\right)^{3/2} & \text{if } |z| < 1\,, \\ 0 & \text{if } |z| \geq 1\,, \end{cases} \tag{15}$$

where $z = 2\phi/D$, $\phi = E - \text{int}(E + 0,5)$ – phase, $E = (t - T_0)/P$ – (non-integer) cycle number, t – time, T_0 – initial epoch, P – period, D – full duration of minimum in units of P.

Papageorgiou et al. (2014) made a statistical study of a sample of eclipsing binaries. They have used an oversimplified approximations of the light curves, approximating the my a parabolic fit over overlapping intervals $[-0.2, +0.2]$, $[0.1, 0.4]$, $[0.3, 0.7]$, $[0.6, 0.9]$, $[0.8, 1.2]$. Obviously, the first and last interval correspond to the same observations. In Figure 6 we show their fit to our sample light curve. One may see a relatively good approximation of the out–of–eclipse part of the light curve, and a bad approximation of the zone of minimum. A better coincidence of the fit near minima may be expected for EW-type stars, whereas for EA-type stars our NAV algorithm produces significantly better approximation for all phases.

To illustrate the dependence of the "best fit" light curves on the "non-linear" parameters $C_8 \ldots C_{11}$, we show corresponding approximations in Fig. 7 and Fig. 8. The thick line in the middle of each figures corresponds to the curve for the sample parameters 1, -0.04, 0.01, -0.05, 0.01, -0.8, -0.6, 0.11, 2, 3.3, 0 for $C_1 \ldots C_{11}$, respectively.

One may see the significant variations of the shape of the curve and, for each real observations, the best fit solution is expected to be unique. As in previous cases, the solution may be determined using different methods.

4 CONCLUSIONS

The "simplified" and "phenomenological" models are discussed. The behaviour of the test functions resembles that of the test-function for the "physical" model based on the Wilson and Devinney (1971) code and its improvements. Few algorithms for the statistically optimal determination of the parameters have been tested on these test functions, and we prefer to use a "combined" algorithm, where the best method for an estimate of the next set of the parameters is chosen at each step, making the convergence of the numerical solution as fast as possible.

The specified shapes – either for the "simplified" model, or the "phenomenological" one – are much more effective for the EA-type stars with narrow minima, but also can be applied to EB-type and EW-type stars with smooth variations.

We developed the software realizing various methods for study of variable stars. The results of this study will be used in the frame of the projects "Ukrainian Virtual Observatory" (UkrVO) (Vavilova et al., 2012) and "Inter-Longitude Astronomy" (Andronov et al., 2010).

REFERENCES

Anderson, T. W. (1984), *An Introduction to Multivariate Statistical Analysis*, Wiley Series in Probability and Mathematical Statistics, Wiley, ISBN 9780471521600, URL `http://books.google.com.pe/books?id=9dA0AAAACAAJ`.

Andronov, I. L. (1991), Structure and evolution of stars, Odessa Inst. Adv. Teachers.

Andronov, I. L. (1994), (Multi-) Frequency Variations of Stars. Some Methods and Results, *Odessa Astronomical Publications*, **7**, p. 49.

Andronov, I. L. (2003), Multiperiodic versus noise variations: mathematical methods, in C. Sterken, editor, *Interplay of Periodic, Cyclic and Stochastic Variability in Selected Areas of the H-R Diagram*, volume 292 of *Astronomical Society of the Pacific Conference Series*, p. 391.

Andronov, I. L. (2010), Mathematical modeling of the light curves using the "new algol variables" (nav) algorithm, Int. Conf. KOLOS-2010 Abstract Booklet.

Andronov, I. L. (2012), Phenomenological modeling of the light curves of algol-type eclipsing binary stars, *Astrophysics*, **55**, pp. 536–550, arXiv: `1208.3655`.

Andronov, I. L., Antoniuk, K. A., Baklanov, A. V., Breus, V. V., Burwitz, V., Chinarova, L. L., Chochol, D., Dubovsky, P. A., Han, W., Hegedus, T., Henden, A., Hric, L., Chun-Hwey, K., Yonggi, K., Kolesnikov, S. V., Kudzej, I., Liakos, A., Niarchos, P. G., Oksanen, A., Patkos, L., Petrik, K., Pit', N. V., Shakhovskoy, N. M., Virnina, N. A., Yoon, J. and Zola, S. (2010), Inter-Longitude Astronomy (ILA) Project: Current Highlights And Perspectives. I. Magnetic vs. Non-Magnetic Interacting Binary Stars, *Odessa Astronomical Publications*, **23**, p. 8.

Andronov, I. L. and Tkachenko, M. G. (2013), Comparative Analysis of Numerical Methods for Parameter Determination, *Czestochowski Kalendarz Astronomiczny 2014*, pp. 173–180, arXiv: `1310.1967`.

Cherepashchuk, A. M. (1993), Parametric Models in Inverse Problems of Astrophysics, *Astronomicheskii Zhurnal*, **70**, p. 1157.

Kallrath, J. and Milone, E. F. (2009), *Eclipsing Binary Stars: Modeling and Analysis*, Springer-Verlag, New York, iSBN 978-1-4419-0698-4.

Korn, G. A. and Korn, T. M. (1968), *Definitions, theorems, and formulas for reference and review*, McGraw-Hill, New York.

Marquardt, D. (1963), A Method for the Solution of Certain Problems in Least Squares, *J. Soc. Indust. Appl. Math*, **11**(2), pp. 431–441.

Papageorgiou, A., Kleftogiannis, G. and Christopoulou, P.-E. (2014), An automated search of O'Connell effect from surveys of eclipsing binaries, *Contributions of the Astronomical Observatory Skalnate Pleso*, **43**, pp. 470–472.

Parenago, P. P. and Kukarkin, B. W. (1936), The Shapes of Light Curves of Long Period Cepheids. Mit 12 Abbildungen., *Zeitschrift für Astrophysik*, **11**, p. 337.

Pickering, E. (1881), Variable Stars of Short Period., *Proc. Amer. Acad. Arts and Sciences*, **16**, pp. 257–278.

Press, W. H., Teukolsky, S. A., Vetterling, W. T. and Flannery, B. P. (2007), *Numerical Recipes 3rd Edition: The Art of Scientific Computing*, Cambridge University Press, New York, NY, USA, 3 edition, ISBN 0521880688, 9780521880688.

Vavilova, I. B., Pakulyak, L. K., Shlyapnikov, A. A., Protsyuk, Y. I., Savanevich, V. E., Andronov, I. L., Andruk, V. N., Kondrashova, N. N., Baklanov, A. V., Golovin, A. V., Fedorov, P. N., Akhmetov, V. S., Isak, I. I., Mazhaev, A. E., Golovnya, V. V., Virun, N. V., Zolotukhina, A. V., Kazantseva, L. V., Virnina, N. A., Breus, V. V., Kashuba, S. G., Chinarova, L. L., Kudashkina, L. S. and Epishev, V. P. (2012), Astroinformation resource of the Ukrainian virtual observatory: Joint observational data archive, scientific tasks, and software, *Kinematics and Physics of Celestial Bodies*, **28**, pp. 85–102.

Wilson, R. E. and Devinney, E. J. (1971), Realization of Accurate Close-Binary Light Curves: Application to MR Cygni, *Astrophys. J.*, **166**, p. 605.

Zola, S., Gazeas, K., Kreiner, J. M., Ogloza, W., Siwak, M., Koziel-Wierzbowska, D. and Winiarski, M. (2010), Physical parameters of components in close binary systems - VII, *Monthly Notices Roy. Astronom. Soc.*, **408**, pp. 464–474.

Zola, S., Kolonko, M. and Szczech, M. (1997), Analysis of a photoelectric light curve of the W UMa-type binary ST Ind., *Astronomy and Astrophysics*, **324**, pp. 1010–1012.