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Confronting models of twin peak quasi-periodic oscillations: Mass and spin estimates fixed by neutron star equation of state

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ABSTRACT
Twin-peak quasiperiodic oscillations (QPOs) are observed in the X-ray power-density spectra of several accreting low-mass neutron star (NS) binaries. In our work we consider several QPO models and focus especially on the atoll source 4U 1636-53 with its large set of QPO measurements. We find that the considered models require the QPO excitation radii in 4U 1636-53 to be close to the inner-most stable circular orbit of the accretion disc. We explore and summarize mass-angular-momentum relations and limits on NS compactness implied by individual QPO models. We confront these relations with NS parameters given by various NS equations of state (EoS). The application of concrete EoS removes the degeneracy in the mass and angular momentum determined from the QPO models when the spin frequency is known. Moreover, the applied NS EoS are compatible only with some of the considered QPO models. In our work we compare simplified calculations that assume Kerr background geometry to the detailed calculations considering NS oblateness influence in Hartle–Thorne spacetimes.

Keywords: X-rays: binaries – Accretion, accretion disks – Stars: neutron – Equation of state

1 INTRODUCTION

Accreting neutron stars (NSs) are believed to be the compact component in more than 20 low mass X-ray binaries (LMXBs). In these systems, the mass is transferred from the companion by overflowing the Roche lobe and forming an accretion disk that surrounds the NS. The disk contributes significantly to high X-ray luminosity of these objects while the most of the radiation comes from its inner parts and disk-NS boundary layer. According to their X-ray spectral and timing properties, the NS LMXBs were further classified into Z and atoll sources, whose names were inspired by the shapes of tracks they trace in the color-color

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diagram (e.g. van der Klis, 2005). While the Z sources are generally more stable and brighter, the atoll sources are weaker and shows significant changes in X-ray luminosity. Both classes exhibit a variability over a large range of frequencies. Apart of irregular changes, their power spectra contain also relatively coherent features known as quasiperiodic oscillations (QPOs).

So called low frequency QPOs have frequencies in the range of 1–100 Hz. In the case of Z-sources they have been further classified into horizontal, flaring, and normal branch oscillations (HBO, FBO and NBO, respectively) according to the position of the source in the color-color diagram. Oscillations of similar properties to HBOs were observed also in several atoll sources (see van der Klis, 2006 for a review). Much attention among theorists is however attracted to the kilohertz QPOs (100–1000 Hz) because their high frequencies are comparable to the orbital timescale in a vicinity of a NS. It is believed that this coincidence represents a strong indication that the corresponding signal originates in the innermost parts of the accretion disks or close to the surface of the NS itself. This believe was also supported by means of Fourier-resolved spectroscopy (e.g. Gilfanov et al., 2000).

The kHz QPOs have similar properties in both Z and atoll sources. They are frequently observed in pairs often called twin peak QPOs. Their 'upper' and 'lower' QPO frequencies (ν_u and ν_l, respectively) exhibit a strong and remarkably stable positive correlation and clustering around the rational ratios. These ratios are emphasized either due to the intrinsic source clustering or weakness of the two QPOs outside the limited frequency range (suggesting possible resonant energy exchange between two physical oscillators Abramowicz et al., 2003a; Belloni et al., 2005, 2007; Török et al., 2008a,b,c; Barret and Boutelier, 2008; Horák et al., 2009; Boutelier et al., 2010). The other properties of each oscillation (e.g. the rms-amplitude and the quality factor) seem to depend mostly on its frequency, and the way how they vary is different between the upper or lower oscillation. These differences often help to identify the type of kHz QPO in cases when only one peak is present in power spectra (Barret et al., 2005, 2006; Méndez, 2006; Török, 2009).

Many models have been proposed to explain the rich phenomenology of twin peak QPOs (Alpar and Shaham, 1985; Lamb et al., 1985; Miller et al., 1998; Psaltis et al., 1999; Wagoner, 1999; Wagoner et al., 2001; Abramowicz and Kluźniak, 2001; Kluźniak and Abramowicz, 2001; Kato, 2001; Titarchuk and Wood, 2002; Abramowicz et al., 2003b,c; Rezzolla et al., 2003; Kluźniak et al., 2004; Pétri, 2005; Zhang, 2005; Bursa, 2005; Török et al., 2007; Kato, 2007, 2008; Stuchlík et al., 2008; Čadež et al., 2008; Kostić et al., 2009; Germanà et al., 2009; Mukhopadhyay, 2009 and several others). While any acceptable model should address both the excitation mechanism and subsequent modulation of the resulting X-ray signal as well as their overall observational properties, most of the theoretical effort has been so far devoted to the observed frequencies. Clearly, their correlations serve as a first test of the model viability.

Comparison between the observed and expected frequencies can reveal the mass and angular momentum of the NS. These can be confronted with models of rotating NS based on a modern equation of state (EoS, e.g. Urbanec et al., 2010b). Here we extend the work started by Török et al. (2010, 2012). We explore and summarize findings on mass-angular-momentum relations and limits on NS compactness implied by several QPO models. We confront these findings with NS parameters given by various EoS. Our paper briefly sketch some results from the prepared publication of Török et al. (2015).

2 TWIN PEAK QPO MODELS AND THEIR APPROXIMATION IN KERR SPACETIMES

Within the framework of many QPO models, the observable frequencies can be expressed directly in terms of epicyclic frequencies. Formulae for the Keplerian, radial and vertical epicyclic frequency in Kerr spacetimes were first derived by Aliev and Galtsov (1981). In a commonly used form (e.g. Török and Stuchlík, 2005) they read

$$\varOmega_{\mathrm{K}} = \frac{\mathcal{F}}{j + x^{3/2}}\,, \quad \nu_{\mathrm{r}} = \varGamma\varOmega_{\mathrm{K}}\,, \quad \nu_{\theta} = \varDelta\varOmega_{\mathrm{K}}\,, \tag{1}$$

where

$$\varGamma = \sqrt{\frac{-3j^2 + 8j\sqrt{x} + (-6+x)\,x}{x^2}}\,, \quad \varDelta = \sqrt{1 + \frac{j\left(3j - 4\sqrt{x}\right)}{x^2}}\,, \tag{2}$$

$x \equiv r/M$, and the "relativistic factor" $\mathcal{F}$ reads $\mathcal{F} \equiv c^3/(2\pi GM)$. We note that Kerr geometry represents an applicable approximation of NS spacetimes when the compact object mass is high (Török et al., 2010; Urbanec et al., 2013).

2.1 Twin peak QPO Models

Here we investigate a subset of models which have been previously considered in studies of Török et al. (2011, 2012). Below we briefly outline the list of these models.

RP model. The relativistic precession model explains the kHz QPOs as a direct manifestation of modes of relativistic epicyclic motion of blobs at various radii r in the inner parts of the accretion disc (Stella and Vietri, 1999). For the RP model, one can easily solve relations defining the upper and lower QPO frequencies in terms of the orbital frequencies to arrive at an explicit formula which relates the upper and lower QPO frequencies in units of Hertz as (Török et al., 2010, 2012)

$$\nu_{\mathrm{L}} = \nu_{\mathrm{U}}\left\{1 - \left[1 + \frac{8j\nu_{\mathrm{U}}}{\mathcal{F} - j\nu_{\mathrm{U}}} - 6\left(\frac{\nu_{\mathrm{U}}}{\mathcal{F} - j\nu_{\mathrm{U}}}\right)^{2/3} - 3j^2\left(\frac{\nu_{\mathrm{U}}}{\mathcal{F} - j\nu_{\mathrm{U}}}\right)^{4/3}\right]^{1/2}\right\}. \tag{3}$$

TD model. Concept similar to RP model where QPOs are generated by a tidal disruption of large accreting inhomogenities (Germanà et al., 2009). The evaluation of the explicit relation between the two observed QPO frequencies is possible in a way similar to the RP model (Török et al., 2012),

$$\nu_{\mathrm{U}} = \nu_{\mathrm{L}}\left\{1 + \left[1 + \frac{8j\nu_{\mathrm{L}}}{\mathcal{F} - j\nu_{\mathrm{L}}} - 6\left(\frac{\nu_{\mathrm{L}}}{\mathcal{F} - j\nu_{\mathrm{L}}}\right)^{2/3} - 3j^2\left(\frac{\nu_{\mathrm{L}}}{\mathcal{F} - j\nu_{\mathrm{L}}}\right)^{4/3}\right]^{1/2}\right\}. \tag{4}$$

WD model. Oscillation model that assumes non-axisymmetric modes (Kato, 2001). The upper and lower QPO frequencies for the WD model can be expressed as

$$\nu_{\mathrm{U}} = 2\,(1 - \varGamma)\,\varOmega_{\mathrm{K}}\,, \quad \nu_{\mathrm{L}} = (2 - \varGamma)\,\varOmega_{\mathrm{K}}\,. \tag{5}$$

RP1 and RP2 models. Models dealing with non-axisymmetric disc-oscillation modes whose frequencies almost coincide with the frequencies predicted by the RP model (Bursa, 2005; Török et al., 2010). For the RP1 model they can be written as

$$\nu_{\mathrm{U}} = \Omega_{\mathrm{K}} \Delta \,, \quad \nu_{\mathrm{L}} = (1 - \Gamma)\, \Omega_{\mathrm{K}} \,, \tag{6}$$

and for the RP2 model as

$$\nu_{\mathrm{U}} = (2 - \Delta)\, \Omega_{\mathrm{K}} \,, \quad \nu_{\mathrm{L}} = (1 - \Gamma)\, \Omega_{\mathrm{K}} \,. \tag{7}$$

3 MASS AND SPIN OF NS IN ATOLL SOURCE 4U 1636-53 (ESTIMATES ASSUMING HIGH NS COMPACTNESS)

Observations of the peculiar Z-source Circinus X-1 display unusually low QPO frequencies. On the contrary, the atoll source 4U 1636-53 displays the twin-peak QPOs at very high frequencies (see the left panel of Fig. 1). In Török et al. (2011, 2012) we have assumed high mass (Kerr) approximation of NS spacetimes and demonstrated that

- For each twin-peak QPO model and source, the model consideration results in a specific relation between the NS mass M and angular-momentum j rather than in their single preferred combination.
- The data of sources displaying high QPO frequencies (or low frequency ratios, e.g. 4U 1636-53) are much more useful for testing the orbital QPO models than the data of sources displaying low QPO frequencies (or high frequency ratios, e.g. Circinus X-1).
- The considered QPO models require the QPO excitation radii in 4U 1636-53 to be close to the inner-most stable circular orbit of the accretion disc (ISCO).
- The inferred mass of NS in 4U 1636-53 is rather high, above 1.8 $M_{\odot}$, when geodesic models are assumed.

For the atoll source 4U 1636-53 there is a good evidence on the NS spin frequency based on the X-ray burst measurements. Depending on the (two- or one-) hot spot model consideration, the NS spin frequency equals either 291 Hz or 582 Hz (Strohmayer and Markwardt, 2002). Thus, one can in principle infer the angular momentum j and remove the M-j degeneracies related to the individual twin-peak QPO models.

3.1 Twin Peak QPO Models vs. NS EoS

Following Török et al. (2012) we calculate χ^2 maps resulting from fitting of the 4U 1636-53 data for various twin-peak QPO models. These maps are compared to the M-j relations calculated from several NS EoS assuming that the spin frequency is either 290 Hz or 580 Hz (depending on the consideration of one or two hot-spot model for X-ray bursts). In our calculations we follow the approach of Hartle (1967); Hartle and Thorne (1968); Chandrasekhar and Miller (1974); Miller (1977); Urbanec et al. (2010a). We assume the following set of EoS:

- SLy 4 (Říkovská Stone et al., 2003).
- APR (Akmal et al., 1998).
- AU-WFF1, UU-WFF2 and WS-WFF3 (Wiringa et al., 1988; Stergioulas and Friedman, 1995).

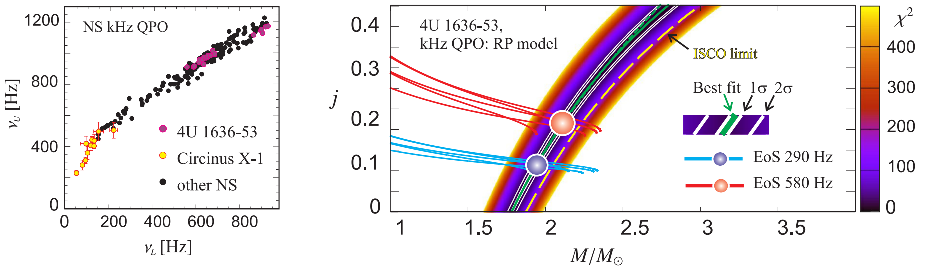

Figure 1. After Török et al. (2012). *Left*: Frequencies of kHz QPOs in various NS sources. *Right*: The χ^2 map of RP model vs. mass-angular momentum relations predicted by NS EoS. The χ^2 map results from the fits of RP model to the kHz QPO data of 4U 1636-53. The *green* line indicates the best χ^2 for a fixed M while the *dashed green* line denotes its quadratic approximation. The *white* lines indicate corresponding 1σ and 2σ confidence levels. The *dashed-yellow* line indicates a simplified estimate on the upper limits on M and j assuming that the highest observed upper QPO frequency in 4U 1636-53 is associated to the ISCO. The NS EoS are assumed for the rotational frequency inferred from the X-ray burst measurements. The *blue* spot roughly indicates the combination of mass and spin resulting from the consideration of the spin frequency 290 Hz, several concrete equations of state and given QPO model. The *red* spot indicates the same but for the spin frequency 580 Hz.

In the right panel of Fig. 1 we illustrate the potential of such approach in the case of the relativistic precession QPO model while other models are considered in Fig. 2. Related indicative estimates of NS parameters are summarized in Table 1.

4 CALCULATIONS IN HARTLE–THORNE SPACETIMES CONSIDERING NS OBLATENESS

So far we have neglected influence of NS oblateness assuming that the star is very compact having thus oblateness factor $\tilde{q} \equiv q/j^2$ close to the Kerr limit, i.e. it has been assumed that $\tilde{q} \sim 1$. In a more general case of $\tilde{q} > 1$, one can assume NS spacetime approximated by Hartle–Thorne geometry (Hartle, 1967; Hartle and Thorne, 1968).

Based on this approximation, the Keplerian orbital frequency can be expressed as (Abramowicz et al., 2003a)

$$\Omega_{\mathrm{K}} = \frac{\mathcal{F}}{x^{3/2}}\left[1 - \frac{j}{x^{3/2}} + j^2 F_1(x) + q F_2(x)\right], \tag{8}$$

where

$$\begin{aligned} F_1(x) &= \left[48 - 80x + 4x^2 - 18x^3 + 40x^4 + 10x^5 + 15x^6 - 15x^7\right] \\ &\quad \left(16(x-2)x^4\right)^{-1} + A(x)\,, \\ F_2(x) &= \frac{5\left(6 - 8x - 2x^2 - 3x^3 + 3x^4\right)}{16(x-2)x} - A(x)\,, \\ A(x) &= \frac{15\left(x^3 - 2\right)}{32} \ln\left(\frac{x}{x-2}\right). \end{aligned}$$

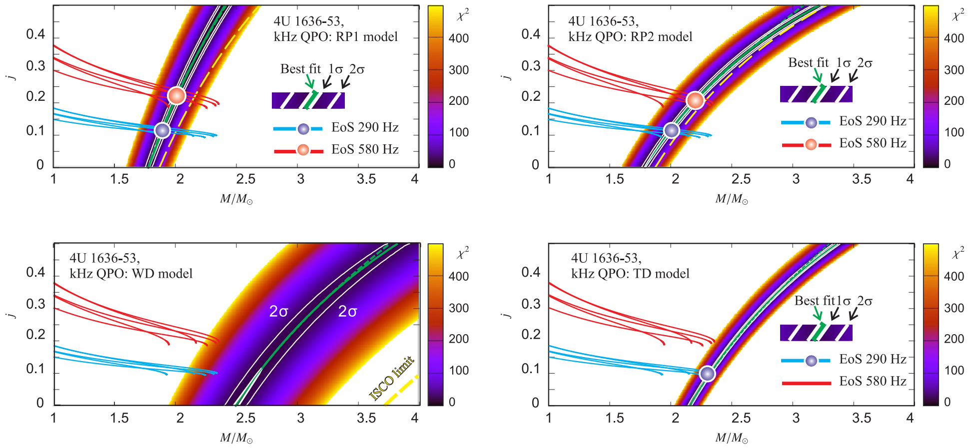

Figure 2. The χ^2 map of RP model vs. mass-angular momentum relations predicted by NS EoS. The *green* line indicates the best χ^2 for a fixed M while the *dashed green* line denotes its quadratic approximation. The *white* lines indicate corresponding 1σ and 2σ confidence levels. The *dashed-yellow* lines indicate simplified estimates on the upper limits on M and j assuming that the highest observed upper QPO frequency in 4U 1636-53 is associated to the ISCO. The NS EoS are assumed for the rotational frequency inferred from the X-ray burst measurements. The *blue* spot roughly indicates the combination of mass and spin resulting from the consideration of the spin frequency 290 Hz, several concrete equations of state and given QPO model. The *red* spot indicates the same but for the spin frequency 580 Hz.

Radial and vertical epicyclic frequency are then described by the following terms

$$\nu_{\mathrm{r}}^2 = \frac{\mathcal{F}^2(x-6)}{x^4}\left[1 + jF_1(x) - j^2F_2(x) - qF_3(x)\right], \tag{9}$$

$$\nu_{\theta}^2 = \frac{\mathcal{F}^2}{x^3}\left[1 - jG_1(x) + j^2G_2(x) + qG_3(x)\right], \tag{10}$$

where

$$\begin{aligned}
F_1(x) &= \frac{6(x+2)}{x^{3/2}(x-6)}\,, \\
F_2(x) &= \left[8x^4(x-2)(x-6)\right]^{-1}\left[384 - 720x - 112x^2 - 76x^3 - \right. \\
&\quad \left. - 138x^4 - 130x^5 + 635x^6 - 375x^7 + 60x^8\right] + A(x)\,, \\
F_3(x) &= \frac{5\left(48 + 30x + 26x^2 - 127x^3 + 75x^4 - 12x^5\right)}{8x(x-2)(x-6)} - A(x), \\
A(x) &= \frac{15x(x-2)\left(2 + 13x - 4x^2\right)}{16(x-6)}\ln\left(\frac{x}{x-2}\right),
\end{aligned} \tag{11}$$

Table 1. Neutron star parameters implied by consideration of twin peak QPO models in Kerr spacetimes. The displayed values result from the confrontation of these models with outcomes of NS modelling shown in Figs. 1 and 2.

Model, frequencies	**M(290 Hz)**	**j(290 Hz)**	**M(580 Hz)**	**j(580 Hz)**
RP $\nu_{\mathrm{L}} = \nu_{\mathrm{K}} - \nu_{\mathrm{r}}$, $\nu_{\mathrm{U}} = \nu_{\mathrm{K}}$	$1.9\,M_{\odot}$	0.11	$2.1\,M_{\odot}$	0.21
TD $\nu_{\mathrm{L}} = \nu_{\mathrm{K}}$, $\nu_{\mathrm{U}} = \nu_{\mathrm{K}} + \nu_{\mathrm{r}}$	$2.3\,M_{\odot}$	0.10	–	–
WD $\nu_{\mathrm{L}} = 2(\nu_{\mathrm{K}} - \nu_{\mathrm{r}})$, $\nu_{\mathrm{U}} = 2\nu_{\mathrm{K}} - \nu_{\mathrm{r}}$	–	–	–	–
RP1 $\nu_{\mathrm{L}} = \nu_{\mathrm{K}} - \nu_{\mathrm{r}}$, $\nu_{\mathrm{U}} = \nu_{\theta}$	$1.8\,M_{\odot}$	0.11	$2.0\,M_{\odot}$	0.21
RP2 $\nu_{\mathrm{L}} = \nu_{\mathrm{K}} - \nu_{\mathrm{r}}$, $\nu_{\mathrm{U}} = 2\nu_{\mathrm{K}} - \nu_{\theta}$	$2.0\,M_{\odot}$	0.11	$2.2\,M_{\odot}$	0.20

$$
\begin{aligned}
G_1(x) &= \frac{6}{x^{3/2}}, \\
G_2(x) &= \left[8x^4(x-2)\right]^{-1}\left[48 - 224x + 28x^2 + 6x^3 - 170x^4 + 295x^5 - \right. \\
&\quad \left. - 165x^6 + 30x^7\right] - B(x), \\
G_3(x) &= \frac{5\left(6 + 34x - 59x^2 + 33x^3 - 6x^4\right)}{8x(x-2)} + B(x), \\
B(x) &= \frac{15(2x-1)(x-2)^2}{16} \ln\left(\frac{x}{x-2}\right).
\end{aligned}
$$

4.1 Results for RP Model (Hartle–Thorne Spacetimes)

Assuming the formulae above we calculated 3D-χ^2 maps for the RP model. In the left panel of Fig. 3 we show behaviour of the best χ^2 as a function of M and j for several color-coded values of $\tilde{q}$. For each value of $\tilde{q}$ there is a preferred M-j relation. We find that, although such a relation has a global minimum, the gradient of χ^2 is always much lower along the relation than the gradient in the perpendicular direction. In other words, χ^2 maps for a fixed $\tilde{q}$ are of the same type as that calculated in Kerr spacetime. It follows then that there is a global M-j-$\tilde{q}$ degeneracy in the sense discussed by Török et al. (2012).

As emphasized by Urbanec et al. (2010b), Török et al. (2010), Kluźniak and Rosińska (2013), Török et al. (2014), and Rosińska et al. (2014), newtonian effects following from the influence of quadrupole moment act on orbital frequencies in opposite way than relativistic

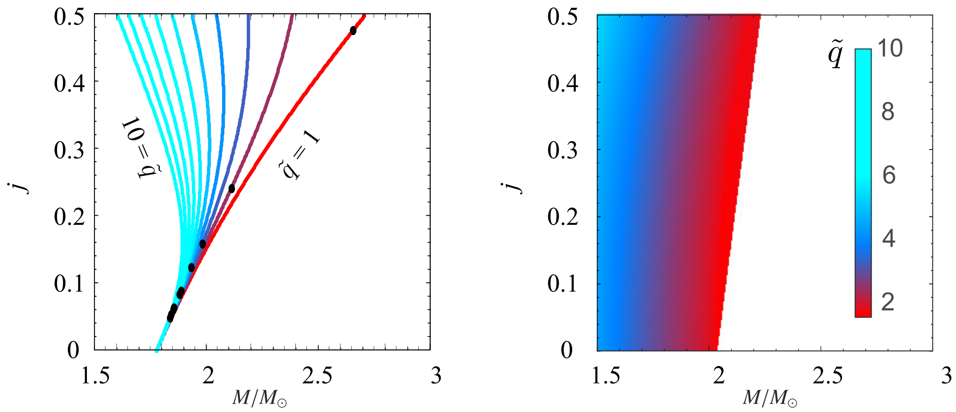

Figure 3. *Left*: Behaviour of the best χ^2 as a function of M and j for several values of $\tilde{q}$. *Dots* denote global minima for each value of $\tilde{q}$ (see however the main text – Section 4.1, for a comment on this). *Right*: The 2D surface in the 3D M-j-$\tilde{q}$ space given by SLy4 EoS.

effects when the angular momentum is increased. The behaviour of the relations shown in the left panel of Fig. 3 is determined by this interplay. Because of this, we can see that high NS oblateness can compensate the increase of the estimated mass due to high angular momentum.

5 CONSIDERATION OF CONCRETE EOS

The relations for RP model drawn in the left panel of Fig. 3 result from fitting of 4U 1636-53 datapoints considering the general Hartle–Thorne spacetime. The consideration does not include strong restrictions following from NS modelling. It can be shown that a concrete NS EoS covers only a 2D surface in the 3D M-j-$\tilde{q}$ space. Thus, when a given EoS is assumed, only corresponding 2D surface is relevant for fitting of datapoints by a given QPO model. Following Urbanec et al. (2013), we illustrate such a surface in the right panel of Fig. 3 for SLy4 EoS. The color-coding of the plot is the same as that on the left panel of the same Figure. The corresponding final M-j-χ^2 map for the RP model is shown in the left panel of Fig. 4. The right panel of this Figure then shows equivalent χ^2 map drawn for the NS mass and spin frequency.

6 CONCLUSIONS

Using Kerr spacetime approximation valid for NS with high compactness (high mass) we find that fitting of twin peak QPO data results rather in mass-angular-momentum (M-j) relations rather than preferred combinations of M and j specific for a given model and source. We also demonstrate that the application of concrete EoS removes the degeneracy in the mass and angular momentum determined from the QPO models when the spin frequency is known. Moreover, the applied NS EoS seem to be compatible only with some of the considered QPO models.

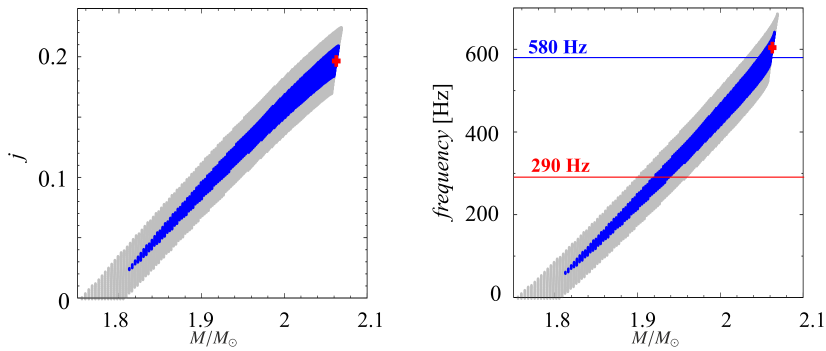

Figure 4. *Left*: The final M-j-χ^2 map for the RP model. Global minimum is denoted by *red* marker. The *dark* colour area denotes 1σ confidence level, the *light* colour area denotes 2σ confidence level. *Right*: The same map, but drawn for the NS spin frequency. The horizontal *blue* line denotes the higher possible spin frequency measured from X-ray bursts (i.e. 580 Hz). The horizontal *red* line denotes the lower possible spin frequency measured from X-ray bursts (i.e. 290 Hz).

Detailed consideration of rotating NS spacetimes including the influence of NS oblateness reveal M-j relations similar to the case of Kerr approximation. Finally, inspecting the left panel of Fig. 4, we can see that the concrete EoS, SLy4, considered for RP model then implies a clear M-j relation. This relation exhibits a shallow minimum. The right panel of the same Figure shows the equivalent relation between the NS mass and spin frequency as well as its shallow minimum. Taking into account the favoured spin frequency inferred from X-ray bursts, 580 Hz, we can see that the NS mass and angular momentum have to be around

$$M \sim 2.05\, M_\odot\,, \qquad j \sim 0.2\,. \tag{12}$$

These values are in good agreement with those inferred from the simplified consideration using Kerr spacetimes given in Table 1. Considering the shallow χ^2 minima denoted in Fig. 4, it can be interesting that its frequency value almost coincides with the measured spin frequency of 580 Hz.

In the Figure 5 we show several relations between the mass and spin frequency obtained for RP model and miscellaneous EoS. These relations are similar to the one discussed above. However, we can see that in several cases given EoS does not provide any match for the spin 580 Hz or even for the spin 290 Hz. This can rule out the considered QPO model and EoS combination. The selection effect comes from the limits on maximal mass allowed by individual EoS. Full discussion of these results will be presented in Török et al. (2015) along with an analogical consideration of the other models examined here in Section 3 and listed in Table 1.

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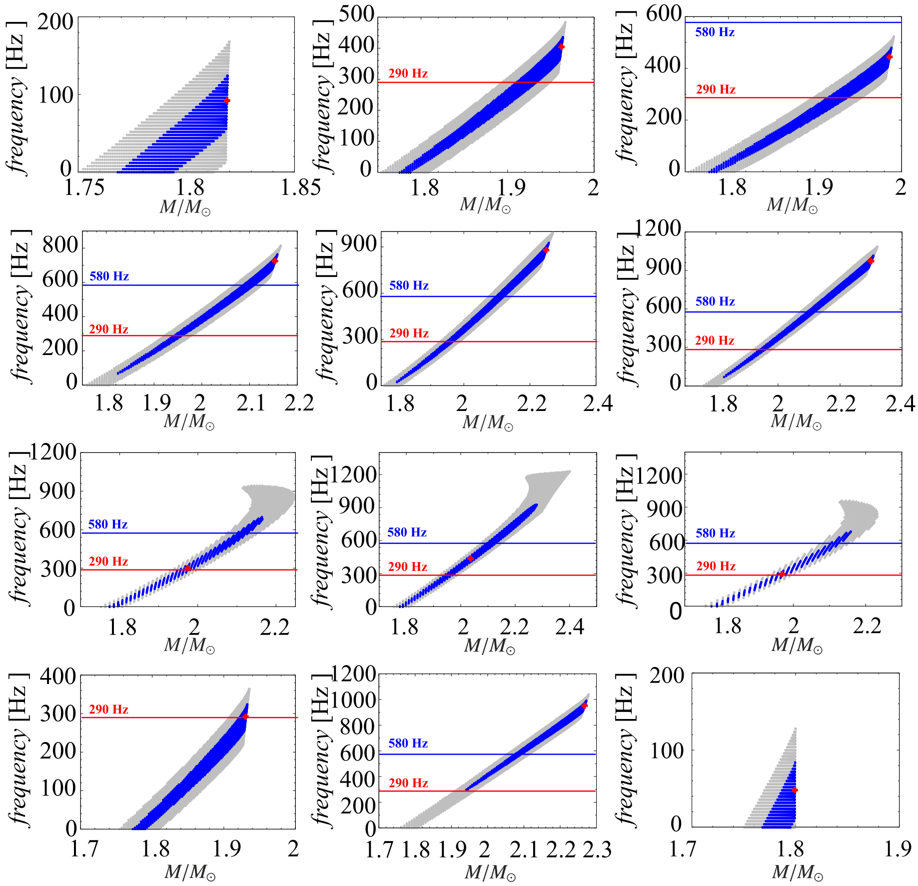

Figure 5. The mass-spin χ^2 maps for the RP model and 12 different EoS. Global minima are denoted by *red* marker. The *dark* colour areas denote 1σ confidence levels, the *light* colour areas denote 2σ confidence levels. The horizontal *blue* lines denote the higher possible spin frequency measured from X-ray bursts (i.e. 580 Hz). The horizontal *red* lines denote the lower possible spin frequency measured from X-ray bursts (i.e. 290 Hz).

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