

# Twin peak quasi-periodic oscillations as signature of oscillating cusp torus

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## ABSTRACT

We propose a new model of twin-peak quasi-periodic oscillations. This model considers an oscillating torus with cusp that changes location of its centre around radii very close to innermost stable circular orbit. Preliminary results of analytically and computationally complex calculations indicate that the model can provide very good fits of data and matches several neutron star equations of state.

**Keywords:** X-rays: binaries – Accretion, accretion disks – Stars: neutron – Equation of state

## 1 INTRODUCTION

Many models have been proposed to explain a phenomenon of twin peak quasi-periodic oscillations observed in neutron-star low-mass X-ray binaries (QPOs in LMXBs). It is believed that QPOs are carrying signatures of strong gravity and dense matter composition. Serious theoretical effort has been devoted to explain the observed frequencies and their correlations. The brief introduction to twin peak QPOs and their models can be found in paper of Török et al. (2014) in this Volume.

One of the first QPO models, the so called relativistic-precession model (RP model) identifies the twin-peak kHz QPO frequencies  $\nu_U$  and  $\nu_L$  with two fundamental frequencies of a nearly circular geodesic motion: the Keplerian orbital frequency and the periastron-precession frequency,

$$\nu_U = \nu_K, \quad \nu_L = \nu_{\text{per}} = \nu_K - \nu_r, \quad (1)$$

where  $\nu_r$  denotes the radial epicyclic frequency. The correlations among them is then obtained by varying the radius of the underlying circular orbit in a reasonable range. Within this framework it is usually assumed that the variable component of the observed X-ray signal originates in a bright localized spot or blob orbiting the neutron star on a slightly eccentric

orbit. The observed radiation is then periodically modulated due to the relativistic effects. It has been shown that the model is roughly matching the observed  $\nu_U(\nu_L)$  correlations (Stella and Vietri, 1999; Belloni et al., 2007; Török et al., 2012). Nevertheless the RP model also suffers some theoretical difficulties. It is not clear whether the modulation of a radiation from a small localized spot can produce sufficiently strong signal modulation to explain a relatively large observed QPO amplitudes. It is then expected that larger spots (giving higher amount of modulated photons) can undergo a serious shearing due to the differential rotation in the surrounding accretion disk. This does not agree with a high coherence of the QPO signal which is often observed. The model also lacks an explanation of inferred existence of preferred orbits which should be responsible for appearance of QPO pairs and clustering of their frequencies.

Only slightly later, Abramowicz and Kluźniak (2001); Kluźniak and Abramowicz (2001) proposed concept of orbital resonance models. Within this concept, QPOs originate in resonances between oscillation modes of the accreted fluid. The most quoted, so called 3:2 epicyclic resonance model identifies the resonant eigenfrequencies with frequencies  $\nu_\theta$  and  $\nu_r$  of radial and vertical epicyclic axisymmetric modes of disc (or torus) oscillations. It is assumed that

$$\nu_U = \nu_\theta, \quad \nu_L = \nu_r \Leftrightarrow \nu_U/\nu_L = 3/2, \quad (2)$$

while the correlation  $\nu_U(\nu_L)$  arises from resonant corrections to the eigenfrequencies (Abramowicz et al., 2005a,b). We stress that the model deals with a collective motion of the accreted matter. Moreover, the oscillation modes of innermost region of the accretion flow can modulate the amount of matter transferred to NS surface through the boundary layer (Paczynski, 1987; Abramowicz et al., 2007; Horák, 2005). Therefore, it may naturally explain both high amplitudes and coherence of the kHz QPOs. Nevertheless, it is questionable whether the resonant corrections to the eigenfrequencies can be large enough to explain the whole observed range of  $\nu_U$  and  $\nu_L$ . Furthermore, it was shown that the model implies large range of NS masses and has difficulties when confronted to models of rotating NS based on up-to-date equations of state (EoS, see Urbanec et al., 2010; Török et al., 2012).

Motivated by partial success of above models and their complementary difficulties, we present a modified framework for interpreting twin peak QPOs. Our paper sketch results from the prepared publication of Török et al. (2015).

## 2 OSCILLATING TORI

Our model is largely based on the theoretical work of Straub and Šrámková (2009). Throughout this Section we adopt Kerr geometry as description of slowly rotating compact NS. We assume that the innermost region of accretion flow is hot enough to form a pressure supported torus of a moderate thickness. Assuming a non-relativistic polytropic equation of state and neglecting the poloidal components of the fluid velocity (so that the fluid follows circular orbits), the equilibrium torus shape and its structure are completely determined by the Lane–Emden function, which is given by a simple analytic formula (Straub and Šrámková, 2009; Abramowicz et al., 2006)

$$f = 1 - \frac{1}{nc_{s0}^2} \ln \frac{\mathcal{E}}{\mathcal{E}_0}. \quad (3)$$

In this equation,  $\mathcal{E} = (-g^{tt} + 2\ell g^{t\phi} - \ell^2 g^{\phi\phi})^{-1/2}$  denotes the energy of a particle on a (non-geodesic) circular orbit having the specific angular momentum  $\ell$ . We assume that the angular momentum is constant in the whole volume of the torus,  $\ell = \ell_0 = \text{const}$ . As we assume that the torus is located in the vicinity of the innermost stable circular orbit (ISCO) where also Keplerian angular momentum is nearly constant, we believe that it is a reasonable approximation. Meaning of other symbols in Eq. (3) is straightforward:  $n$  is the polytropic index ( $n = 3$  for a radiation pressure dominated fluid),  $g^{\mu\nu}$  are the contravariant component of Kerr metric (we employ the  $(-+++)$  signature) and  $c_{s0}$  is the sound speed at the center of the torus located at radius  $r_0$  in the equatorial plane, where the pressure gradient vanishes and where the energy  $\mathcal{E}$  takes the value  $\mathcal{E}_0$ . Vanishing of the pressure forces in the torus center implies that the streamline  $r = r_0, \theta = \pi/2$  is a geodesic line and therefore the fluid angular momentum takes the Keplerian value at that radius,  $\ell_0 = \ell_K(r_0)$ .

The surfaces of constant density and pressure coincide with those of constant  $f$  and their values can be calculated from  $f$  by  $\rho = \rho_0 f^n$  and  $p = p_0 f^{n+1}$ , where  $\rho_0$  and  $p_0$  refer to the values at the torus center that corresponds to  $f = 1$ . On the other hand, the surface of the torus, where both pressure and density vanishes is given by the condition  $f = 0$ . It is also worth to note that the position of the center  $r_0$  and a shape of these surfaces are entirely given by the value of  $\ell_0$  and the spacetime geometry, while the particular values of  $p$  and  $\rho$  and therefore also the location of the overall surface of the torus are set by the central value of the sound speed  $c_{s0}$ .

Straub and Šrámková (2009) introduce a dimensionless parameter  $\beta$  that characterizes a size of the torus,

$$\beta = \frac{\sqrt{2n}c_{s0}}{r_0\mathcal{E}_0(\ell_0 g_0^{\phi\phi} - g_0^{t\phi})}. \quad (4)$$

This parameter is roughly proportional to the Mach number of the flow at the torus center as can be seen from its Newtonian limit  $\beta = \sqrt{2n}(c_s/r\Omega)_0$  (compare with Blaes, 1985). In addition, it is also roughly proportional to the ratio of the radial (or vertical) extension of the torus to its central radius  $r_0$ . Hence, the sound-crossing time and the dynamical timescale of the torus are roughly similar.

## 2.1 Marginally overflowing tori (cusp tori)

The stationary solution does not exist for an arbitrary large value of  $\beta$  (Abramowicz et al., 1978). Apart of the obvious limit  $\beta \leq 1$ , there is much stronger constrain coming from general relativity. Large enough tori that extend below the ISCO radius, may be terminated there by a “cusp”, where the rotation of the flow becomes Keplerian again. This is a consequence of the fact that the Keplerian angular momentum close to a relativistic object reaches its minimum at ISCO and raises up again bellow.

The cusp corresponds to a saddle point of the Lane–Emden function and the corresponding self-crossing equipotential limits the surface of any stationary rotating fluid configuration with given angular momentum  $\ell_0$ . Fluid that appear outside this surface, is accreted onto the central star on the dynamical timescale driven by gravity and pressure forces without need of viscosity Paczynski (1977). Abramowicz et al. (1978) calculated

analytically the accretion rate from a slightly overflowing torus, his result agrees very well with numerical simulations.

The critical value of the  $\beta$ -parameter giving a marginally overflowing torus follows from Eqs. (3) and (4),

$$\beta_c(r_0) = \frac{\sqrt{2 \ln(\mathcal{E}_c/\mathcal{E}_0)}}{r_0 \mathcal{E}_0 (\ell_0 g_0^{\phi\phi} - g_0^{t\phi})}, \quad (5)$$

where  $\mathcal{E}_c = \mathcal{E}(r_c)$  is the particle energy at the cusp. Its location  $r = r_c$  can be found by equating the Keplerian angular momentum to the fluid angular momentum  $\ell_0$ . This procedure leads to the third-order algebraic equation (in  $\sqrt{r_c}$ ), giving the position of the cusp in terms of the location of the torus center,

$$r_c^{3/2} - \frac{(2r_0^{1/2} - jM^{1/2})(r_0^{1/2} - jM^{1/2})}{(r_0^{3/2} - 2Mr_0^{1/2} + jM^{3/2})M^{1/2}} (r_c - r_0^{1/2}r_c^{1/2}) + j \frac{r_0(r_0^{1/2} - j)}{r_0^{3/2} - 2Mr_0^{1/2} + jM^{3/2}} = 0, \quad (6)$$

where  $r_0 \geq r_{\text{ISCO}}(j)$ . If the stellar spin is neglected ( $j = 0$ ), this equation is reduced to the quadratic one and its solution can be expressed as

$$r_c = r_0 \left( \frac{M + \sqrt{(2r_0 - 3M)M}}{r_0 - 2M} \right)^2, \quad r_0 \geq 6M \quad (7)$$

and the critical  $\beta$ -parameter reads

$$\beta_c = \frac{(r_0 - r_c)(r_0 - 2M)^2 [r_0 r_c - 2M(r_0 + 2r_c)]^{1/2}}{r_c r_0 (r_c - 2M)^{1/2} (r_0 - 3M)^{1/2}}. \quad (8)$$

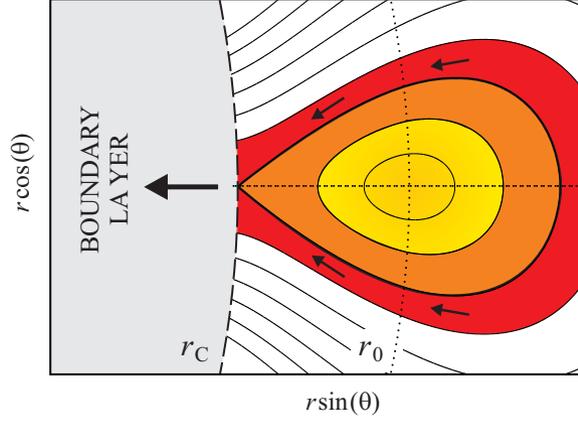
## 2.2 Frequencies of epicyclic oscillations

Abramowicz et al. (2006) pointed out the existence of the radial and vertical epicyclic modes that describes a global motion of the torus. They have found that, in the limit of infinitesimally slender tori  $\beta \rightarrow 0$ , frequencies of this modes  $\nu_R$  and  $\nu_V$  measured in the fluid reference frame coincide with the epicyclic frequencies of test particles,

$$\nu_r = \left( 1 - \frac{6M}{r} + \frac{8jM^{3/2}}{r^{3/2}} - \frac{3j^2M^2}{r^2} \right)^{1/2} \nu_K, \quad (9)$$

$$\nu_\theta = \left( 1 - \frac{4jM^{3/2}}{r^{3/2}} + \frac{3j^2M^2}{r^2} \right)^{1/2} \nu_K, \quad (10)$$

while at fixed azimuth their frequencies are given by  $\nu_{R,m} = \nu_r + m\nu_K$  and  $\nu_{V,m} = \nu_\theta + m\nu_K$  with  $m$  being the integer azimuthal wave number. In particular, the  $m = -1$  radial and



**Figure 1.** Illustration of the equipotential surfaces of an accretion torus. The yellow colour denotes a non-accreting equilibrium torus. The orange colour denotes the case of the cusp torus.

vertical modes give the frequencies of the periastron and nodal precession of a weakly eccentric and tilted torus. It is also worth to note that they now describe a collective motion of the fluid, rather than a motion of individual particles.

In a more realistic case, when  $\beta \geq 0$ , the pressure gradients start to contribute to the restoring force of the perturbed torus shifting their frequencies to new ‘corrected’ values,

$$\nu_{R,m}(r_0, \beta) = \nu_r(r_0) + m\nu_K(r_0) + \Delta\nu_{R,m}(r_0, \beta), \quad (11)$$

$$\nu_{V,m}(r_0, \beta) = \nu_\theta(r_0) + m\nu_K(r_0) + \Delta\nu_{V,m}(r_0, \beta). \quad (12)$$

The pressure corrections  $\Delta\nu_{R,m}$  and  $\Delta\nu_{V,m}$  have been calculated by Straub and Šrámková (2009) using perturbation expansion in  $\beta$ -parameter. They found that a first non-zero corrections are of the order of  $\beta^2$ .

### 3 FREQUENCY IDENTIFICATION

We identify the observed QPO frequencies with frequencies of the epicyclic modes of torus oscillations. We propose that the upper kilohertz QPO frequency is the Keplerian orbital frequency of the fluid at the center of the torus, where both pressure and density peaks and from which the most of torus radiation emerges. The lower kilohertz QPO corresponds to the frequency of the non-axisymmetric  $m = -1$  radial epicyclic mode. Overall, there is

$$\nu_U \equiv \nu_K(r_0), \quad \nu_L \equiv \nu_{R,-1}(r_0, \beta). \quad (13)$$

The QPO frequencies are then strong functions of the position of the center of the torus  $r_0$  and its thickness  $\beta$ . Obviously, a choice  $\beta = 0$  (slender tori) recovers the RP model frequencies completely, as the QPO frequencies would be now given entirely by the geodesic frequencies. In addition, in the case of a finite thickness  $\beta > 0$ , they also weakly depend on the value of the polytropic index  $n$ . In the following discussion, we fix  $n = 3$  as the inner parts of the accretion flow are believed to be radiation-pressure dominated.

We assume the cusp configuration

$$\beta(r_0) \doteq \beta_c(r_0). \quad (14)$$

In other words, we expect that for given  $r_0$  is the torus always close to its maximal possible size, just filling its ‘Roche-like’ lobe.

Thus, our model predicts that QPO frequencies are function of single parameter, the position of the center of the torus  $r_0$ ,

$$\nu_u \equiv \nu_K(r_0), \quad \nu_l \equiv \nu_{R,-1}[r_0, \beta_c(r_0)]. \quad (15)$$

Therefore, one obtains a unique correlation among them by changing this parameter in a reasonable range. In the next section we compare this predicted correlation with the data of the atoll source 4U 1636-53.

#### 4 APPLICATION TO TWIN PEAK QPOS IN 4U 1636-53

Török et al. (2012, 2014) have confronted several QPO models to the data of atoll source 4U 1636-53. They have outlined a comparison between individual matches of the model to the data as well as quantitative estimates of inferred NS parameters. We apply the same fitting procedure to the discussed cusp torus model.

##### 4.1 Non-rotating approximation

First, we investigate the case of a simple one parametric fit assuming non-rotating NS approximated by Schwarzschild geometry. In this way we can obtain a comparison to the RP model and a rough estimate of the NS mass implied by our cusp torus model.

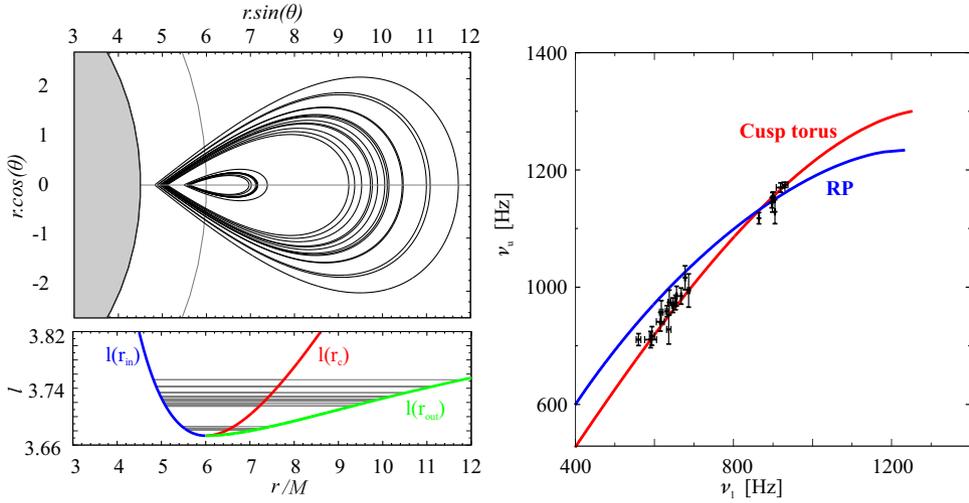
In the left panel of Fig. 2 we plot the sequence of equipotential contours of cusp tori which provides the best match of 4U 1636-53 data. In the right panel of the same Figure we show this best fit. The RP model best fit is included for comparison. Clearly, the cusp torus model matches the observed trend better than the RP one. In more detail, the related  $\chi^2$  improvement is about  $\Delta\chi^2 \approx 80\%$ . The NS mass inferred from the cusp torus model is then

$$M_0 = 1.69 [\pm 0.01] M_\odot, \quad (16)$$

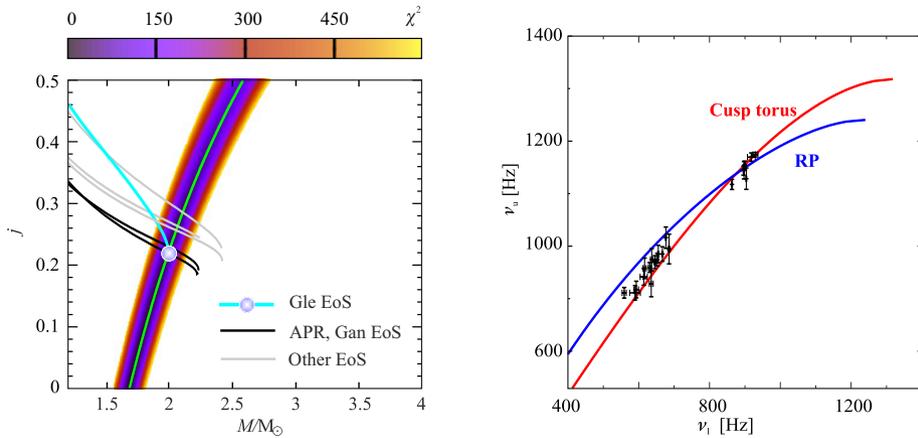
where the scatter in the estimated mass corresponds to the  $2\sigma$  confidence level. Considering results of Török et al. (2012, 2014), we can expect that the mass (16) belongs to a mass-angular momentum relation implied by the model.

##### 4.2 Consideration of NS rotation

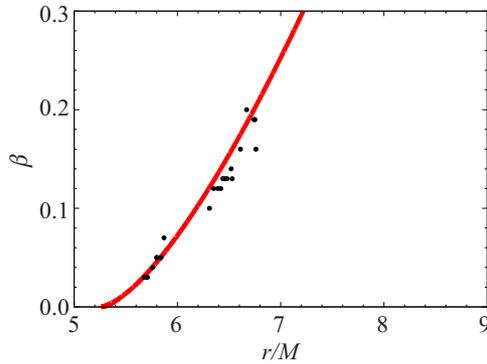
The results of the two-dimensional fitting of the parameters  $M$  and  $j$  are shown in the left panel of Fig. 3. The Figure illustrates  $\chi^2$  behaviour in the form of color-coded map. Remarkably, the best fits are reached when  $M$  and  $j$  are related through the specific relation denoted by the green line.



**Figure 2.** *Left:* Sequence of cusp tori corresponding to one-parametric fit of 4U 1636-53 data. *Bottom* panel indicates angular momentum behaviour. *Right:* Corresponding frequency relation (red curve) plotted together with the data. The blue curve indicating the best fit by RP model ( $j = 0$ ) is shown for a comparison.



**Figure 3.** *Left:* Two-dimensional color-coded map of  $\chi^2$  behaviour resulting from fitting of data points by the cusp torus model. The green curve indicates the preferred mass-angular momentum relation. The other curves indicate mass-angular momentum relations predicted by models of rotating NS. These are drawn for several NS and spin (rotational frequency) 580 Hz inferred from the X-ray burst measurements. The spot roughly indicates combination of  $M \sim 2 M_\odot$  and  $\sim 0.2$ . *Right:* Consideration of  $j = 0.22$ . The red curve indicate the prediction of cusp torus model. The blue curve indicates the best fit by RP model for the same angular momentum,  $j = 0.22$ .



**Figure 4.** Combinations of  $\beta$  and  $r$  exactly matching individual data points vs. cusp torus relation.

## 5 DISCUSSION AND CONCLUSIONS

There is good evidence on the NS spin frequency of 4U 1636–53 based on X-ray burst measurements. Depending on the (two- or one-) hot-spot model consideration, the spin  $\nu_s$  reads either  $\nu_s \doteq 290$  Hz or  $\nu_s \doteq 580$  Hz (Strohmayer and Markwardt, 2002). The value of 580 Hz is usually preferred. In the left panel of Fig. 3 we include several mass-angular momentum relations expected from models of rotating NS (see Török et al., 2014 for details) assuming this spin. We can see that there are overlaps between these relations and the relation inferred from the cusp torus model.

In the right panel of Fig. 3 we show the best fit of the model to the data for  $j = 0.22$  corresponding to

$$M_0 = 2.00 [\pm 0.02] M_\odot, \quad (17)$$

where the scatter in the estimated mass corresponds to the  $2\sigma$  confidence level. We choose  $j = 0.22$  as a referential value since it roughly corresponds to three different EoS. Furthermore, as discussed by Urbanec et al. (2013), the NS oblateness factor is decreasing along the displayed EoS relation towards the low values close to the Kerr limit. Thus, the Kerr approximation adopted here should be well applicable. In the same panel, the RP model best fit drawn for  $j = 0.22$  is included for a comparison. In analogy to the non-rotating case, the cusp torus fit is better than a fit based on RP model. Having these results we also attempted to fit the data by the discussed torus frequencies but considering any torus thickness and fixed  $M = 2 M_\odot$  and  $j = 0.22$ . We searched for the combinations of  $\beta$  and  $r$  matching each individual data point. The result of this procedure is shown in Fig. 4. Clearly, the obtained values are distributed very close to the cusp relation.

Overall, there is a strong indication that twin peak QPOs can be identified with a particular non-axisymmetric  $m = -1$  radial epicyclic mode and Keplerian orbital motion associated to the cusp torus. These modes may naturally give strong modulation of both emerging radiation and the accretion rate. They are therefore very good candidates for explaining high amplitudes of QPO. In addition, their eigenfrequencies change only weakly on the spatial scale of the turbulent motion, therefore it may be expected that they may survive also in highly turbulent media typical for accretion flows.

Finally, we note that the presented concept has also potential to explain the observed low frequency QPOs. As noticed by Rosińska et al. (2014); Kluźniak and Rosińska (2013) the frequencies of vertical epicyclic modes seem to be very sensitive to the NS quadrupole moment. Their consideration thus rather exceeds the framework of Kerr spacetime approximation adopted here. Nevertheless, we roughly investigated also the frequencies of non-axisymmetric  $m = -1$  vertical epicyclic mode of cusp tori. Assuming the same mass, angular momentum and radii as those in Figs. 3 and 4 we obtained values of tens of Hertz. These are of the same order as the observed frequencies of low frequency QPOs. The  $m = -1$  vertical epicyclic mode may therefore play the same role in the framework of cusp torus model as the Lense–Thirring precession in the framework of RP model.

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