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The Komissarov Model of Sgr A*

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ABSTRACT
Astrophysical black holes observed from Earth have a very small apparent size in the sky. The largest of all is the supermassive black hole, Sagittarius A*, in the centre of our Galaxy with an apparent diameter of 53 μas (micro-arcseconds). We construct a model of magnetized accretion torus surrounding the central black hole in Sgr A* in two different geometries of the magnetic field. For the toroidal one ("the Komissarov model") we assume stationarity, axial symmetry, constant specific angular momentum, polytropic equation of state and small optical depth. The last assumption allows one to use the ray-tracing technique to calculate the transfer of radiation. For the mass and spin of the Sgr A* black hole we adopt $M = 4 \times 10^6\, M_\odot$ and $a = 0.5$.

Keywords: Galaxy: centre – Accretion, accretion discs – Black hole physics – Relativistic processes

1 INTRODUCTION

Sagittarius A* was first observed in the radio band (Balick and Brown, 1974), but its observed emission ranges from radio to X-ray energies. The most remarkable feature of Sgr A* is its complex variability at all observable wavelengths. The luminosity fluctuations increase with increasing wave energy, from a factor of a few at radio to a few orders of magnitude in the X-ray band (see e.g. Genzel et al., 2010, for a review). The spectral peak lies in the millimetre radio band and brings forth a peak luminosity of $\lesssim 10^{36}$ erg$\cdot s^{-1}$. The accretion structure around Sgr A* is thus extremely dim given its enormous mass. Therefore, adequate disc models describe a radiatively inefficient emitter like an advection dominated accretion flow (ADAF, Narayan and Yi 1995) or an ion torus (see Straub et al., 2012, and references therein). The term *advection* means here that a large part of the gravitationally liberated thermal energy is not converted into radiation but carried inward with the ionised, hot accretion flow.

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In the millimetre radio range, i.e. at wavelengths corresponding to the spectral peak of Sgr A*, the Event Horizon Telescope (EHT), operational in 2015–2020, and the orbital telescope RadioAstron, launched in 2011, will be able to perform high resolution Very Large Baseline Interferometry (VLBI) observations. Like this, images of the immediate environment of the black hole will be obtained, in particular those of the accretion flow.

These new observational possibilities on Sgr A* have stimulated a lot of recent research, reviewed, e.g. by Broderick et al. (2014). The hope is that a detailed knowledge of theoretically predicted observational appearance of the structure of the accretion flow in Sgr A* will provide powerful and reliable tools to test Einstein's general relativity at its strong field limit. While eventually sophisticated numerical models (Global General Relativistic Radiation Magnetohydrodynamics – GGRRMHD etc.) of Sgr A* will be used to make a meaningful comparison between theory and observations, in the foreseeable future simple *analytic* models will be invaluable as a secure guide in the vast parameter space that needs to be explored.

Following this idea, we have recently constructed an analytic optically thin Polish Doughnut model of Sgr A* (Straub et al., 2012). It assumed that the magnetic field in Sgr A* had no global structure, but instead was chaotic, i.e. locally isotropic. In this paper we make the next logical step by considering a model with a globally ordered (toroidal) magnetic field. We use the Komissarov (2006) analytic model of a magnetized optically thin Polish Doughnut and follow all its assumptions. In the Komissarov model, all general relativistic effects, and influence of the (toroidal) magnetic field are fully and exactly taken into account. They are calculated from the first principles with no approximation. The presence of a magnetic field is important in calculations of the synchrotron radiation emissivity, which is also done. We consider here a torus-shaped, barotropic, and *stationary* disk with axisymmetry and constant angular momentum around a Kerr black hole. The disk is fully ionized. These assumptions reflect the basic physics of the real object.

We summarize the basic features of the magnetized torus model and its synchrotron radiation in Sections 2 and 3, respectively. Section 4 presents conclusions and perspectives.

2 MAGNETIZED ACCRETION TORUS

2.1 Toroidal magnetic field (the Komissarov model)

We constructed a magnetized accretion torus at the Galactic centre using the model developed by Komissarov (2006), which describes analytically a polytropic accretion torus with toroidal magnetic field in the Kerr spacetime. The magnetized accretion flow is described by the conservation law that takes the form,

$$\nabla_\alpha T^\alpha_\beta = 0\,, \qquad T^\alpha_\beta = \left(h + b^2\right) u^\alpha u_\beta + \left(p + \frac{b^2}{2}\right) \delta^\alpha_\beta - b^\alpha b_\beta\,, \tag{1}$$

where $h = p + \rho$ is the enthalpy, with p and ρ being the pressure and energy density of the fluid, and b is the magnitude of the magnetic field. The induction equation is

$$\nabla_\alpha \left({}^*F^{\alpha\beta}\right) = 0\,, \qquad {}^*F^{\alpha\beta} = u^\alpha b^\beta - b^\alpha u^\beta \tag{2}$$

and the equation of continuity

$$\nabla_\alpha (n u^\alpha) = 0\,. \tag{3}$$

The fluid 4-velocity is assumed to be

$$\mathbf{u} = \left(u^t, 0, 0, u^\varphi\right), \tag{4}$$

using Boyer–Lindquist coordinates. We assume a constant specific angular momentum

$$\ell_0 \equiv -u_\varphi / u_t\,. \tag{5}$$

This quantity is expressed in terms of the dimensionless specific angular momentum

$$\lambda = \frac{\ell_0 - \ell_{\rm ms}}{\ell_{\rm mb} - \ell_{\rm ms}}, \quad 0 \le \lambda \le 1\,, \tag{6}$$

where $\ell_{\rm ms}$ and $\ell_{\rm mb}$ are the specific angular momentum at the marginally stable and bound orbits respectively. These assumptions fully determine the 4-velocity.

The gas and magnetic pressures p and p_m are assumed to follow the polytropic prescription

$$p = \kappa h^{\rm k}\,, \qquad p_m = \kappa_m \mathcal{L}^{\rm k-1} h^{\rm k}\,, \tag{7}$$

where κ and κ_m are polytropic constants, k is the polytropic index (assumed identical for gas and magnetic pressures), and $\mathcal{L} \equiv g_{t\varphi}^2 - g_{tt} g_{\varphi\varphi}$ where $g_{\mu\nu}$ is the Kerr metric.

The conservation of stress-energy leads to

$$W_{\rm s} - W + \frac{\rm k}{\rm k-1}\left(\kappa + \kappa_m \mathcal{L}^{\rm k-1}\right) h^{\rm k-1} = 0 \tag{8}$$

where the potential $W = -\ln|u_t|$ is used. We assume that the torus fills its Roche lobe, which fixes the central radius of the torus and its surface, thus the values of the potential at the centre, $W_{\rm c}$, and at the surface, $W_{\rm s}$, of the torus. This immediately gives

$$h = h_{\rm c} \left(\omega \frac{\kappa + \kappa_m \mathcal{L}_{\rm c}^{\rm k-1}}{\kappa + \kappa_m \mathcal{L}^{\rm k-1}}\right)^{1/({\rm k}-1)}\,, \tag{9}$$

where $\omega = (W - W_{\rm s})/(W_{\rm c} - W_{\rm s})$ and $\mathcal{L}_{\rm c}$ is the value of $\mathcal{L}$ at the centre of the torus.

The polytropic constants κ and κ_m can be expressed according to

$$\kappa = h_{\rm c}^{1-\rm k} (W_{\rm c} - W_{\rm s}) \frac{\rm k-1}{\rm k} \frac{\beta_{\rm c}}{1+\beta_{\rm c}}\,, \qquad \kappa_m = \frac{\mathcal{L}_{\rm c}^{1-\rm k}}{\beta_{\rm c}} \kappa\,, \tag{10}$$

where $\beta_{\rm c}$ is the central magnetic pressure ratio, $\beta_{\rm c} \equiv p_{\rm c}/p_{m,{\rm c}}$. The electron number density

$$n_{\rm e} = \frac{h - \kappa h^{\rm k}}{\mu_{\rm e} m_u} \tag{11}$$

is then known analytically.

The magnetic field in Boyer–Lindquist frame is assumed to be toroidal: $b^\alpha = (b^t, 0, 0, b^\varphi)$. Using the definition of the magnetic pressure, $p_m = b^\alpha b_\alpha/2$, and assuming that $b^\alpha u_\alpha = 0$ (i.e. the magnetic field 4-vector is in the rest space of the comoving observer), it is easy to get

$$b^\varphi = \sqrt{\frac{2p_m}{g_{\varphi\varphi} + 2\ell_0 g_{t\varphi} + \ell_0^2 g_{tt}}}, \qquad b^t = \ell_0 b^\varphi , \tag{12}$$

which is fully known analytically.[1] The magnitude of the magnetic 3-vector field $\boldsymbol{B}$ measured by an observer comoving with the fluid is then $||\boldsymbol{B}|| = \sqrt{b^\alpha b_\alpha}$. Let us consider one synchrotron photon emitted at a given point of the torus. Let $\boldsymbol{p}$ be the 4-vector tangent to the photon geodesic and $\boldsymbol{l}$ be its projection orthogonal to $\boldsymbol{u}$. The angle ϑ between the magnetic field $\boldsymbol{B}$ and the direction of emission is given by $\boldsymbol{l} \cdot \boldsymbol{B} = ||\boldsymbol{l}||\,||\boldsymbol{B}||\cos\vartheta$, it is known analytically as well.

We now note that such an accretion torus cannot be made of a perfect gas. If it were, then $pm_u/(\rho k_\mathrm{B}) = T/\mu_\mathrm{e}$ where T is the electron temperature and k_B is the Boltzmann constant. However, it is easy to see that p/ρ is independent of the central value of the enthalpy h_c. Thus the temperature would be independent of h_c as well, and would be purely determined by the geometry of spacetime, which does not make sense. We will still assume that there exists a relation $T = \mathrm{C}p/\rho$ where C is a constant, but does not take its perfect-gas value. Rather, we choose T_c at the centre of the torus and define the constant C by $T_\mathrm{c} = \mathrm{C}p_\mathrm{c}/\rho_\mathrm{c}$. Then

$$T = T_\mathrm{c}\left(\frac{\rho}{\rho_c}\right)^{\mathrm{k}-1} \tag{13}$$

depends on the choice of T_c, and no longer only on spacetime geometry.

2.2 Isotropic magnetic field

An accretion torus with isotropic (i.e. chaotic) magnetic field has already been studied around Sgr A* by Straub et al. (2012). This model is simply the limit of the Komissarov (2006) model with $\kappa_m = 0$. The section above thus directly applies to this simpler case. The magnetic field strength is obtained by assuming that the magnetic pressure is everywhere related to the gas pressure through

$$p_m = \frac{1}{\beta}p \tag{14}$$

thus, the β parameter is valid in the whole torus, not only at its centre. Then the magnetic field strength is given by

$$B^2 = 2p_m . \tag{15}$$

[1] Note that b^t is proportional to b^φ is a natural result of setting $b^r = 0$.

3 SYNCHROTRON RADIATION

Millimetre-wavelength emission coming from Sgr A* has been attributed to a region close to the event horizon of the supermassive black hole and can be explained by thermal synchrotron radiation (see, e.g. Genzel et al., 2010).

3.1 Toroidal magnetic field

Wardziński and Zdziarski (2000) show that for a mildly relativistic Maxwellian electron distribution,

$$n_{\mathrm{e}}(\gamma) = \frac{n_{\mathrm{e}}}{\theta_{\mathrm{e}}} \frac{\gamma\left(\gamma^2-1\right)^{1/2}}{\mathrm{K}_2\left(1/\theta_{\mathrm{e}}\right)} \exp\left[-\frac{\gamma}{\theta_{\mathrm{e}}}\right], \tag{16}$$

where $\theta_{\mathrm{e}} = k_{\mathrm{B}}T/(m_{\mathrm{e}}c^2)$, m_{e} being the electron mass, $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorenz factor and K_2 is a modified Bessel function, the emission coefficient is

$$j_\nu^{\mathrm{dir}} = \frac{\pi e^2}{2c}(\nu\nu_0)^{1/2}\mathcal{X}(\gamma_0)n_{\mathrm{e}}(\gamma_0)\left(1+2\frac{\cot^2\vartheta}{\gamma_0^2}\right)\times$$
$$\times\left[1-\left(1-\gamma_0^{-2}\right)\cos^2\vartheta\right]^{1/4}\mathcal{Z}(\vartheta,\gamma_0), \tag{17}$$

where $\nu_0 \equiv eB/(2\pi m_{\mathrm{e}}c)$ is the cyclotron frequency. The superscript dir means that this emission coefficient depends on the angle between the magnetic field and the direction of emission, no angle averaging has been performed. Then,

$$\gamma_0 = \begin{cases} \left[1+\left(\frac{2\nu\theta_{\mathrm{e}}}{\nu_0}\right)\left(1+\frac{9\nu\theta_{\mathrm{e}}\sin^2\vartheta}{2\nu_0}\right)^{-\frac{1}{3}}\right]^{\frac{1}{2}} & \theta_{\mathrm{e}} \lesssim 0.08\,, \\ \left[1+\left(\frac{4\nu\theta_{\mathrm{e}}}{3\nu_0\sin\vartheta}\right)^{\frac{2}{3}}\right]^{\frac{1}{2}} & \theta_{\mathrm{e}} \gtrsim 0.08 \end{cases} \tag{18}$$

is the Lorenz factor of those thermal electrons that contribute most to the emission at ν, and

$$\mathcal{X}(\gamma) = \begin{cases} \left[\frac{2\theta_{\mathrm{e}}(\gamma^2-1)}{\gamma(3\gamma^2-1)}\right]^{1/2}, & \theta_{\mathrm{e}} \lesssim 0.08 \\ \left(\frac{2\theta_{\mathrm{e}}}{3\gamma}\right)^{1/2}, & \theta_{\mathrm{e}} \gtrsim 0.08 \end{cases} \tag{19}$$

$$t \equiv \left(\gamma^2-1\right)^{\frac{1}{2}}\sin\vartheta\,,\ n \equiv \frac{\nu\left(1+t0\left(\gamma_0\right)^2\right)}{\nu_0\gamma},\ \mathcal{Z}(\vartheta,\gamma) = \left\{\frac{t\exp\left[\left(1+t^2\right)^{-\frac{1}{2}}\right]}{1+\left(1+t^2\right)^{\frac{1}{2}}}\right\}^{2n}. \tag{20}$$

Synchrotron radiation becomes self-absorbed below a critical frequency ν_c where the plasma becomes optically thick. The emitted spectrum below this frequency will follow the Rayleigh–Jeans emission law. Following Narayan and Yi (1995) we determine the self-absorption critical frequency by asking that at this frequency and at the current radius r, the synchrotron emission over the volume of the sphere with radius r equates the Rayleigh–Jeans emission from the surface of this sphere, i.e.

$$\frac{4}{3}\pi r^3 j_\nu^{\mathrm{dir}}(\nu_c) = \pi B_\nu(\nu_c)\, 4\pi r^2 \,, \tag{21}$$

where $B_\nu = 2k_B T \nu^2/c^2$ is the Rayleigh–Jeans emission law. The synchrotron emitted spectrum is then smoothly connected with a Rayleigh–Jeans spectrum below ν_c.

3.2 Isotropic magnetic field

Wardziński and Zdziarski (2000) give the angle-averaged limit of the synchrotron emission

$$j_\nu^{\mathrm{avg}} = \frac{2^{1/6}\pi^{3/2}e^2 n_e \nu}{3^{5/6} c \mathrm{K}_2(1/\theta_e) \upsilon^{1/6}} \exp\left[-\left(\frac{9\upsilon}{2}\right)^{1/3}\right] , \tag{22}$$

where $\upsilon = \nu/(\nu_0 \theta_e^2)$. The superscript avg now refers to an emission coefficient after angle averaging. There is a multiplicative correction factor a in this expression in Wardziński and Zdziarski (2000), which is close to unity, and which we do not take into in this work as we are not interested in very precise values of synchrotron fluxes. We note that the equation above is only valid for a mildly relativistic plasma, $\theta_e \lesssim 1$. This condition is satisfied in our model.

The self-absorbed synchrotron is treated in the same way as for the toroidal magnetic field case.

Figure 1 illustrates the synchrotron images of a Komissarov torus at 1.3 mm and demonstrates that for some values of spin and inclination, the size of the emitting zone satisfies the VLBI constraints imposed by Doeleman et al. (2008).

4 CONCLUSION AND PERSPECTIVES

We have constructed a millimetre-wavelength synchrotron radiative model for Sgr A* based on the fully general relativistic, analytical magnetized torus model of Komissarov (2006), who assumes a purely toroidal magnetic field. Our work presented here is only theoretical. It will be useful to construction of a multi-dimensional network of Komissarov's models. For each model of the network one calculates the theoretical silhouette (in different wavelengths) and theoretical electromagnetic spectra of Sgr A*, as could be found in another paper of the same authors (Vincent et al., 2014). This may be then compared with the observed silhouette and spectra. The best fit may give an estimate the black hole spin, and other parameters.

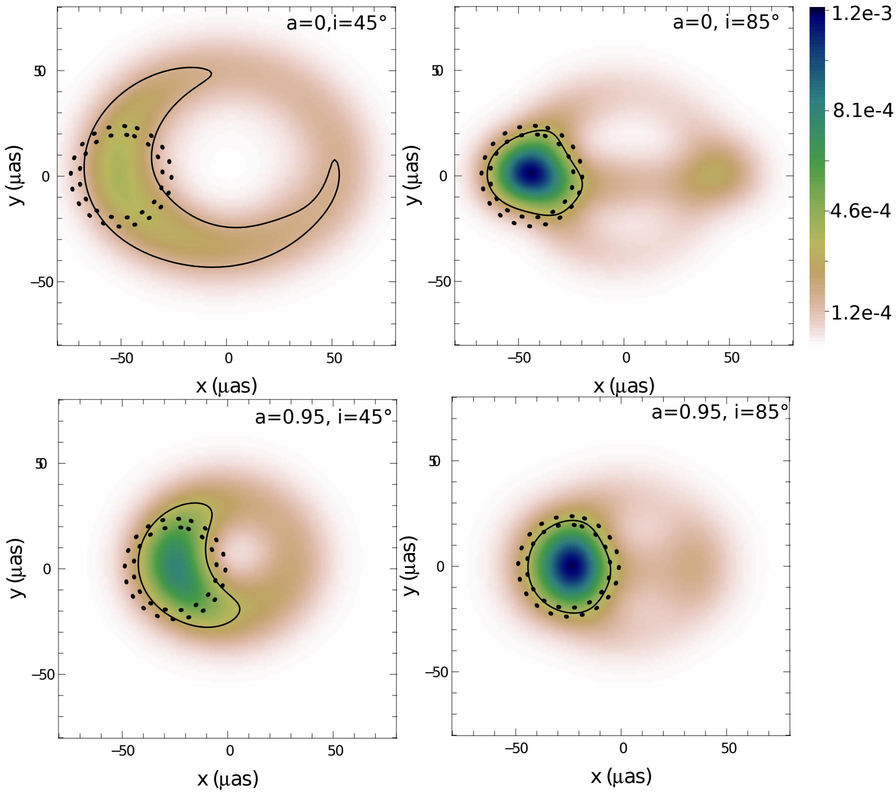

Figure 1. Images (maps of specific intensity) at 1.3 mm of a torus satisfying the millimetre spectral constraints for Sgr A* (see Vincent et al., 2014, for details). The dotted circle show the 1σ confidence domain from Doeleman et al. (2008). The thin solid curve encompass the region containing 50 % of the total flux. Here a and i are the black hole spin and inclination. The color bar is common to all panels, and graduated in cgs units (erg $\cdot$ s^{-1} cm^2 str^{-1} Hz^{-1}).

We believe that the Komissarov model for Sgr A* as developed in this work may be of interest for the future data analysis linked with the EHT project. In particular, this model may be a suitable test bed for investigating the observational counterparts of compact objects alternative to the Kerr black hole.

In order to further test the possibility to constrain the magnetic field geometry in the vicinity of Sgr A*, future work will be dedicated to investigating the polarization predictions of the Komissarov and chaotic models. Future work will also be dedicated to developing a full analysis of the parameter space of the Komissarov model in order to provide a robust fit to observed data.

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