Hysteresis behavior of shocks in low angular momentum flows

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ABSTRACT
In this work we present the GRMHD 1D simulations of accreting matter with variable angular momentum. We focus on the existence and behaviour of the shock in the flow. We show that the location of the shock front responds to the change of the angular momentum, which also causes the accretion rate onto the black hole to vary on different time scales. We study the possible hysteresis behaviour of the shock front during the time evolution of the flow. We discuss the potential observational effects of this phenomenon.

Keywords: black hole – microquasars – accretion flows – shocks

1 INTRODUCTION

During the last decades, the observational data keep confirming the existence of extremely powerful sources of energy ranging on a very broad mass, power, and time scales. Although initially sounding surprisingly, these events are now being theoretically explained as the products of mass accretion on compact objects – neutron stars and black holes. The accretion of matter can proceed in several different regimes which leave observational traces in the measured spectra and lightcurves and which are accompanied by some interesting phenomena, like e.g. jets.

The matter essentially flows into black hole with the speed of light, while the sound speed at maximum can reach $c/\sqrt{3}$. Hence, every accretion flows must have transonic nature. The viscous accretion flow with transonic solution based on the alpha-disk model were first studied by Paczyńsky and Bisnovatyi-Kogan (1981) and Muchotrzeb and Paczynski (1982). After that, Abramowicz and Kato (1989) examined the stability and structure of transonic disks. The possibility of collimation of jets by thick accretion tori was proposed by, e.g., Sikora and Wilson (1981).

In respect of the value of angular momentum there are two main regimes of accretion, the Bondi accretion, which refers to spherical accretion of gas without any angular momentum, and the disk-like accretion with Keplerian distribution of angular momentum. If we consider the same polytropic index and sound speed at infinity of the gas, then in the case of the former, the sonic point is located far away from the compact object and the flow is

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supersonic downstream of it. In the latter case, the flow becomes supersonic quite close to the compact object. For gas with a low value of constant angular momentum, hence belonging in between these two regimes, the equations allow for the existence of two sonic points of both types.

The possible existence of shocks in low angular momentum flows connected with the presence of multiple critical points in the phase space has been studied from different points of view during the last thirty years. However, the theoretical works which describe the fundamental properties of the low angular momentum accretion flow and which usually treat the steady solution of the equations, has so far been carried only for non-magnetized and usually also non-viscous flows. Quasi-spherical distribution of the gas endowed by constant specific angular momentum λ and the arisen bistability was studied already by Abramowicz and Zurek (1981). Later, the significance of this phenomenon related to the variability of some X-ray sources has been pointed out by Jufu and Abramowicz (1988) and soon after, the possibility of the shock existence together with shock conditions in different types of geometries was discussed by Abramowicz and Chakrabarti (1990). More recently, the shock existence was found also in the disc-like structure with low angular momentum in hydrostatic equilibrium both in pseudo-Newtonian potential Das (2002) and in full relativistic approach Das and Czerny (2012). Regarding the sequence of steady solutions with different values of specific angular momentum, the hysteresis-like behaviour of the shock front was proposed by the latter.

Further development of this topic includes numerical simulations of low angular momentum flows in different kinds of geometrical setup. Hydrodynamical models of the low angular momentum accretion flows have been studied already in two and three dimensions, e.g. by Proga and Begelman (2003), Janiuk et al. (2008) and Janiuk et al. (2009). In those simulations, a single, constant value of the specific angular momentum was assumed, while the variability of the flows occurred due to e.g. non-spherical or non-axisymmetric distribution of the matter. The level of this variability was also dependent on the adiabatic index. However, these studies have not concentrated on the existence of the standing shocks as predicted by the theoretical works mentioned above.

Recently, we have carried out simulations of one-dimensional hydrodynamical model of shocked quasi-spherical accretion flow, which confirm the shape of the steady solution and also the dependence of this solution on the leading parameters (angular momentum λ, energy ϵ and adiabatic index γ). These simulations were held in the pseudo-Newtonian framework using the computational code ZEUS (Stone and Norman, 1992; Hayes and Norman, 2003).

The simulations yield the shock front unstable for a subset of parameters which leads to oscillation of the shock front around the position given by the steady solution with frequency depending on the distance to the center (Suková and Janiuk, 2015). Since the shock front is thought to be the source of radiation, this mechanism could be connected with the QPOs with evolving frequency reported from several microquasars (e.g. GX 339-4 (Nandi et al., 2012) or XTE J1550-564 (Chakrabarti et al., 2009)). Moreover, we also show the evolution of the flow with changing angular momentum and reported the repeating creation and disappearance of the shock front due to the hysteresis loop. In another words, we have seen for the first time, that during the dynamical evolution of the accretion flow, the shock was repeatedly created and accreted.

In this work we are reexamining this issue with the general relativistic hydrodynamical simulations done in the Kerr spacetime with both non-spinning and spinning black hole.

2 STANDING SHOCKS IN FLOWS WITH LOW CONSTANT ANGULAR MOMENTUM

In the previous work we studied the behaviour of shocks in low angular momentum flows in the pseudo-Newtonian framework using the Paczynski-Wiita potential to mimic the strong gravity effects near the black hole (Suková and Janiuk, 2015). Because we will use the semi-analytical solution as the initial conditions for our GRMHD simulations, we will shortly repeat here the main equations and derivation.

We assume one dimensional quasi-spherical flow of polytropic gas (with EOS of the form $p = K\rho^{\gamma}$), which posses constant angular momentum λ and accretes with constant accretion rate $\dot{M}$. From the continuity equation we can express the mass accretion rate in the form

$$\dot{M} = u\rho r^2 = \text{const}, \tag{1}$$

where u is the inward radial speed of the gas and ρ is its density.

Further, we can write the energy conservation for the steady state in the form

$$\epsilon = \frac{1}{2}u^2 + \frac{a^2}{\gamma - 1} + \frac{\lambda^2}{2r^2} + \Phi(r), \tag{2}$$

where a is the local sound speed and the term $\Phi(r) = -\frac{1}{2(r-1)}$ represents the Paczynski-Wiita gravitational potential (Paczyńsky and Wiita, 1980), so that r is given in the units of $r_g = 2GM/c^2$.

From these equations we can derive the relation for the radial gradient of the flow velocity

$$\frac{\mathrm{d}u}{\mathrm{d}r} = \frac{\frac{\lambda^2}{r^3} - \frac{\mathrm{d}\Phi(r)}{\mathrm{d}r} + \frac{2a^2}{r}}{u - \frac{a^2}{u}} = \frac{\frac{\lambda^2}{r^3} - \frac{1}{2(r-1)^2} + \frac{2a^2}{r}}{u - \frac{a^2}{u}}. \tag{3}$$

Our aim is to describe the smooth flow, which is subsonic far away from the compact object and which accretes inward onto the compact object. Therefore, at the point, where the denominator of equation (3) equals to zero, the numerator also has to be equal to zero. We call such points the critical points, which have to satisfied the two conditions

$$u_c = a_c, \tag{4}$$

$$a_c = \sqrt{\frac{r_c}{2}\frac{\mathrm{d}\Phi(r_c)}{\mathrm{d}r_c} - \frac{\lambda^2}{2r_c^2}} = \sqrt{\frac{r_c}{4(r_c-1)^2} - \frac{\lambda^2}{2r_c^2}}. \tag{5}$$

We can see that in our settings the critical points coincide with the location of the possible sonic points in the flow (the points, where the radial inward speed equals to the local sound speed $u = a$, hence the radial Mach number $\mathfrak{M} = u/a = 1$). Using these relations, the position of the critical point can be found by solving the equation

$$\epsilon - \frac{\lambda^2}{2r_c^2} + \frac{1}{2(r_c-1)} - \frac{\gamma+1}{2(\gamma-1)}\left(\frac{r_c}{4(r_c-1)^2} - \frac{\lambda^2}{2r_c^2}\right) = 0, \tag{6}$$

After we have found the position of the critical point together with values of $u(r_c)$ and $a(r_c)$, we can integrate equation (3) downwards and upwards and obtain the whole solution.

Important property of the set of equations is that for a subset of parameters, there exist more critical points. In fact, there are three critical points, from which two are of a saddle type and one is of a center type. The topology of the solution in space spanned by r and $\mathfrak{M}$ for spinning black hole was studied by Das and Czerny (2012) and differs for different values of parameters. They showed, that there is set of parameters, for which two global solutions are possible. In this case we have two branches of solution, which passes through the inner and outer saddle critical point. The transonic solution required by the aforementioned boundary conditions, which is the solution having low subsonic velocity far away from the compact object ($\mathfrak{M} < 1$) and supersonic velocity very near to the horizon ($\mathfrak{M} > 1$), thus can locally pass only through the outer or both sonic points. The latter is globally achieved due to the shock formation between the two critical points. So, either the accretion proceeds through the outer sonic point and it is supersonic downwards to the black hole (we call this solution the "Bondi-like type", because the profile of radial Mach number is similar to the Bondi case) or shock can appear at a given radius r_s and connects the two branches, while the entropy accretion rate is increased at the shock location. We call this subspace of the parameter space the shock-existence region.

The possibility of the shock existence is given by the requirement, that the Rankine-Hugoniot shock conditions, which express the conservation of mass, energy and momentum at the shock front are satisfied at some radius r_s. We can write them in the form

$$\frac{\left(\frac{1}{\mathfrak{M}_{\mathtt{in}}} + \gamma\mathfrak{M}_{\mathtt{in}}\right)^2}{\mathfrak{M}_{\mathtt{in}}^2(\gamma-1)+2} = \frac{\left(\frac{1}{\mathfrak{M}_{\mathtt{out}}} + \gamma\mathfrak{M}_{\mathtt{out}}\right)^2}{\mathfrak{M}_{\mathtt{out}}^2(\gamma-1)+2}, \tag{7}$$

which has to hold at the radius r_s. Here, $\mathfrak{M}_{\mathtt{out}}$ and $\mathfrak{M}_{\mathtt{in}}$ is the radial Mach number of the preshock and postshock gas, respectively.

Previously, we run the simulations of the accretion flow behaviour in this framework using the code ZEUS (Suková and Janiuk, 2015) and we confirmed the existence of the stationary shocks, as well as we reported the oscillations of the shock front for lower energies and higher angular momentum of the flow. Later we repeated the study with the GRMHD simulation in the Kerr spacetime (Suková et al., 2016; Suková et al., 2017) using the computational code `Einstein Toolkit` (Löffler et al., 2012) and HARMPI[1] (Ressler et al., 2015, 2017; McKinney et al., 2012; Tchekhovskoy et al., 2007; Mignone and McKinney, 2007) based on the original HARM code (Gammie et al., 2003; McKinney and Gammie, 2004; Noble et al., 2006; Janiuk et al., 2013; Janiuk, 2017). We confirmed the qualitative findings of the earlier study and we pointed out some quantitative differences.

[1] e.g., see `https://github.com/atchekho/harmpi`

In this work we focus on the issue of the hysteresis behaviour of the flow with the GRMHD 1D simulations using the code HARMPI.

3 SIMULATION SETUP

For our GRMHD simulations we set the initial conditions according to the semi-analytical solution (eqns (3) and (6)) with the shock front connecting the two branches placed at an arbitrary position. This solution is not the steady state solution, but the flow quickly settles itself into the real solution after short transient time at the beginning of the computation. For higher values of spin of the black hole a, we use different λ^s for the semi-analytical solution and different λ_g, which governs the rotation of the flow, because the shock-existence region exists for much lower values of angular momentum of the gas.

To study the hysteresis behaviour, we have to change at least one of the leading parameters across the shock-existence region. Physically meaningful is the change of the angular momentum of the incoming matter, which can happen due to the change of conditions in the neighborhood of the black hole, e.g. winds from orbiting stars, matter from magnetic trap held by magnetic companion, gas cloud which passes around the black hole etc. can have slightly different angular momentum in different regions.

The exact time dependence of the angular momentum at the outer boundary depends on the particular physical situation, which causes this change. However, we can assume, that for some time the angular momentum will increase or decrease, that such change will be slow and that the monotonic trend has to stop at some point. We are also interested in the case, when the angular momentum will change periodically (e.g. as a consequence of the orbital or rotational motion of the companion) to see the impact of such behaviour on the flow. Hence, we choose angular momentum as the leading parameter and we run the simulations, in which the angular momentum of the gas incoming through the outer boundary is changed periodically in time according to simple relation

$$\lambda(t) = \lambda(0) - A\sin(t/P), \tag{8}$$

where P is the period and A is the amplitude of the oscillations.

The fiducial value of the polytropic index used in our computations is $\gamma = 4/3$.

4 RESULTS

We run several 1D simulations with changing angular momentum of the incoming gas. We follow the position of the shock and also the mass accretion rate onto the black hole. In fact, we are looking on the mass flux through the inner boundary of our grid, which is placed bellow the horizon of the black hole.

At first we can confirm our earlier results, which show, that the choice between the Bondi-like type of accretion and accretion flow with shock depends on the history of the flow and especially on the presence of the inner sonic point in the flow. Hence we showed that both of the solutions are plausible solutions, which can exist under certain circumstances, and the accretion flow can switch between them during the dynamical evolution of the system.

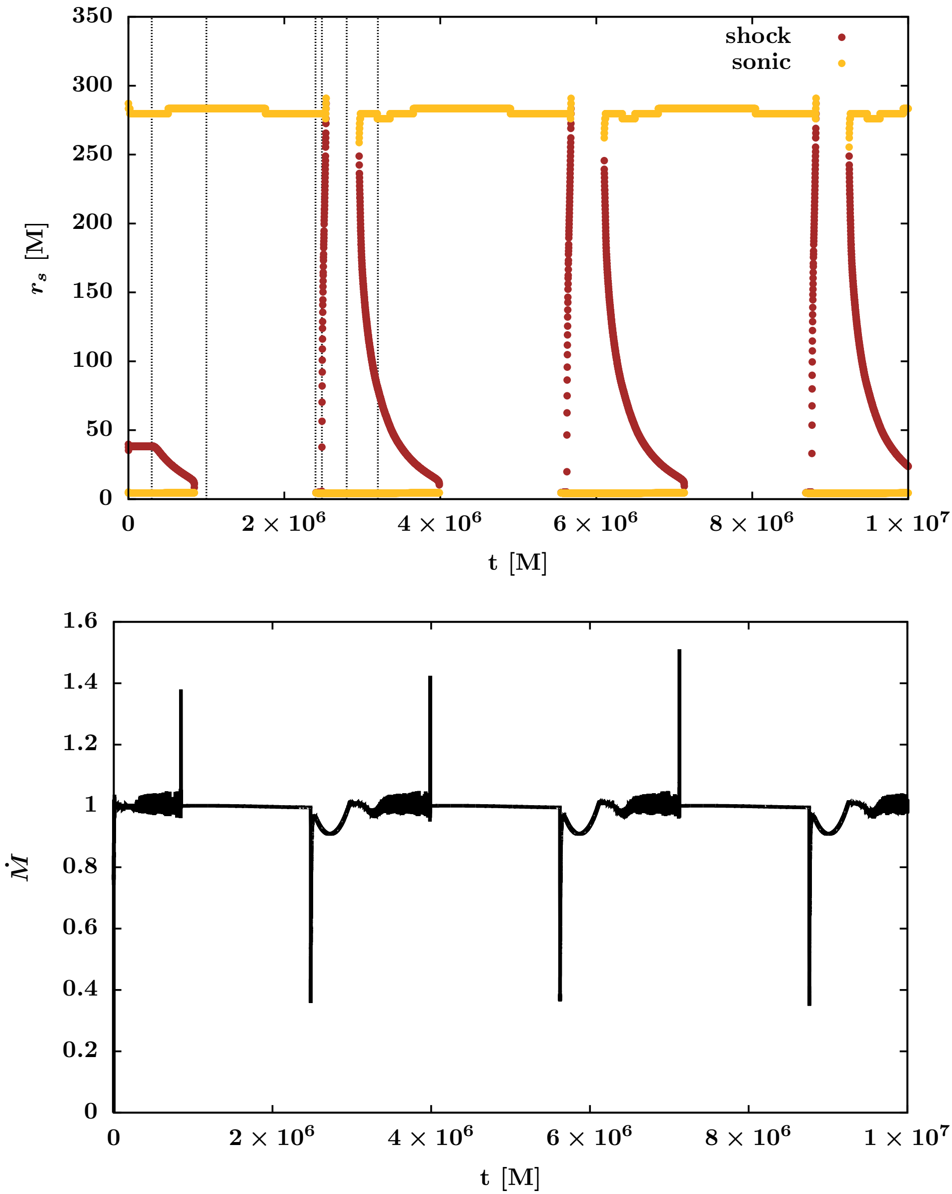

Figure 1. Top panel: Location of the sonic points and the shock front in the flow during the simulation. The vertical lines show the times of the snapshots shown in Fig. 2. Bottom panel: Time dependence of the mass accretion rate through the inner boundary. Parameters of the simulation are $\lambda(0) = 3.55\mathrm{M}$, $\epsilon = 0.0025$, $A = 0.12\mathrm{M}$, $P = 5 \cdot 10^5\mathrm{M}$.

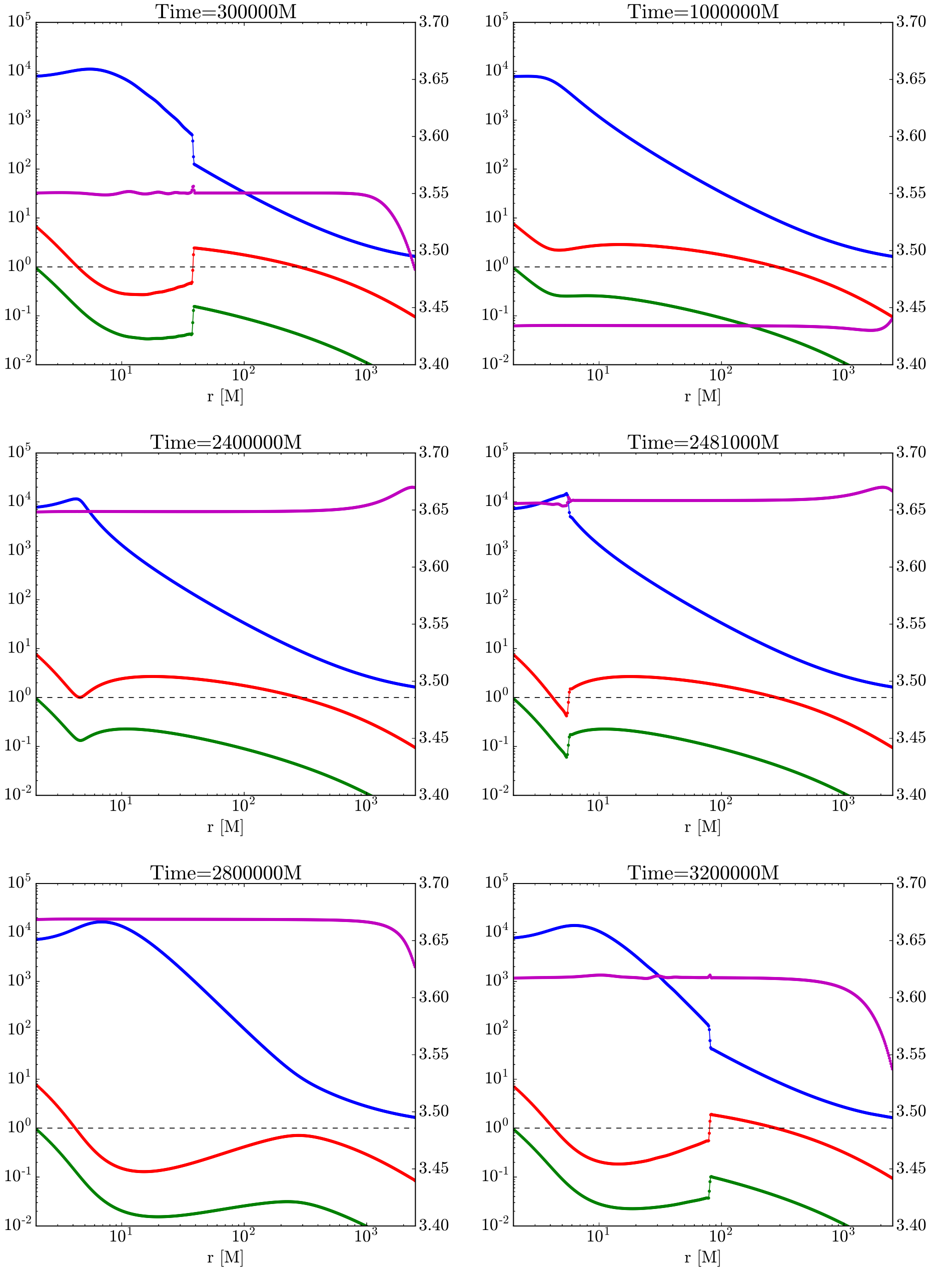

Figure 2. Snapshots from the simulation showing profiles of radial velocity of the gas $u^r_{\rm BL}$ (green) in units of c, Mach number $\mathfrak{M} = -u^r_{\rm BL}/a$ (red) and density ρ in arbitrary units (blue) on the left y-axis and the angular momentum λ [M] (purple) on the right y-axis. Parameters of the simulation are $\epsilon = 0.0025$, $\lambda(0) = 3.55$M, $A = 0.12$M, $P = 5 \cdot 10^5$M.

In Fig. 1 we can see three full hysteresis cycles of the flow and in Fig. 2 six snapshots taken during the first cycle are shown. We can see, that after the change of the angular momentum propagates inside from the outer boundary of the grid, the shock location starts to slowly decrease (first panel in Fig. 2, $t = 3 \cdot 10^5$M). At the moment, when the shock front reaches the minimal stable shock location, it is accreted on the advection time scale. Afterwards, the flow follows the Bondi-like type of solution, where only the outer sonic point exists (second panel in Fig. 2, $t = 10^6$M). When the angular momentum increases back to the shock-existence region, the shock does not form, only the profile of Mach number bends towards $\mathfrak{M} = 1$ in the innermost part of the accretion flow due to the centrifugal barrier (third panel in Fig. 2, $t = 2.4 \cdot 10^6$M). However, at the point, where the angular momentum increases above the shock-existence region, hence the outer sonic point does not exist and the Bondi-like type of solution is not possible anymore, the shock front detaches from the inner sonic point and very quickly expands outwards towards the outer sonic point, with which it merges (fourth panel in Fig. 2, $t = 2.481 \cdot 10^6$M). After that the flow follows the disc-like type of accretion with only the inner sonic point present (fifth panel in Fig. 2, $t = 2.8 \cdot 10^6$M), until the moment, when the angular momentum returns back into the shock-existence region. At that moment shock is created close to the outer sonic point and moves slowly inward (depending on the rate of the angular momentum change given by A and P) and the hysteresis cycle is completed (sixth panel in Fig. 2, $t = 3.2 \cdot 10^6$M).

Simultaneously with the evolution of the shock front position we can also notice the changes in mass accretion rate onto the black hole. When the shock is building in the flow, the radial inward speed decreases and the mass accretion rate drops down. While the shock front location is changing, we can notice some oscillations of the mass accretion rate. At the moment of the abrupt accretion of the shock front from the minimal stable shock position sharp peak occurs in the mass accretion rate. Because the mass accretion rate is a proxy to the luminosity of the source, we can speculate that partially the variability, which we observe in accreting black hole systems, can be induced by the change of the angular momentum of the accreting matter. Considering that these shocks exist in gas with low angular momentum, which gains high energy while falling on the compact object, this conclusion holds especially for the variability in harder spectral bands, above the energy corresponding to the disc black body radiation.

To show the generality of our findings, we performed a set of simulations with different spin of the black hole. Parameters of the runs are $\epsilon = 0.0025, P = 4 \cdot 10^4$M; A) $a = 0, \lambda(0) = 3.55$M$, A = 0.13$M; B) $a = 0.3, \lambda(0) = 3.3M, A = 0.15$M; C) $a = 0.5, \lambda(0) = 3.1M, A = 0.13$M; D) $a = 0.8, \lambda(0) = 2.67M, A = 0.13$M.

In Fig. 3 the mass accretion rate for the four different spins is given. In all cases, we can see similar features, which are the sharp peaks when the shock front is accreted and the decrease of accretion rate followed by oscillations and a broad peak. The time scale of the broad features are given by our chosen rate of change of the angular momentum (P and A). The amplitude of the oscillations and the overall variability is higher for higher spin of the black hole. However, the time scale of the sharp peak is determined by the position of the minimal stable shock position, which is given mostly by value of the polytropic index γ (see Suková and Janiuk (2015) for details in pseudo-newtonian framework), and it does depend on other parameters only slightly. That is illustrated on the second panel of Fig. 3, where we shift the time of each run such that the peak occurs at $t = 1000$M. The height of the peak increases with increasing spin of the black hole, but the time profile is very similar in every case. Duration of the peak is of the order of few hundreds of M, which is in agreement with the X-ray flares seen from Sgr A* (Nowak et al., 2012).

Our last example in Fig. 4 shows the behaviour of the gas with lower energy $\epsilon = 0.0005$. The three runs are set with $P = 2 \cdot 10^5$M; A) $a = 0, \lambda(0) = 3.66\text{M}, A = 0.23\text{M}$; B) $a = 0.3, \lambda(0) = 3.36\text{M}, A = 0.25\text{M}$; C) $a = 0.5, \lambda(0) = 3.05\text{M}, A = 0.33\text{M}$. We adjusted the time in such a way, that one cycle of the hysteresis loop is shown and that the broad peaks coincide. The overall evolution during the cycle agrees with the above described behaviour. The main difference from the previous example is the higher amplitude of the peaks and dips and the fact, that the broad peak is now higher than the sharp peak. The shape and time scale of the sharp peak is however very similar as for the higher energy, hence this feature seems to be quite universal for wide range of parameters.

5 CONCLUSIONS

In the present work we examined the behaviour of the accretion flow with changing angular momentum of the incoming matter. We studied the hysteresis cycle of the shock front location and the induced variability of the accretion rate onto the black hole. In comparison with the previous results, we provided the simulations for spinning black hole with wide range of spins of the black hole. We also varied the value of the energy of the gas. We noted the similarities and differences of the individual runs.

Our numerical results shows that the change of the angular momentum of the matter far away from the black hole (at the outer boundary of our grid) can induce the movement of the shock location in the flow. This can provide the explanation of the shock front motion, which is fitted from the change of the low frequency QPOs seen in some microquasars during the onset and decline of their outburst with the phenomenological propagating oscillatory shock (POS) model (Chakrabarti et al., 2008, 2009; Nandi et al., 2012; Debnath et al., 2013; Debnath et al., 2014).

In case, that the angular momentum crosses the boundary of the shock-existence region in the parameter space (keeping other parameters constant) the shock front can disappear suddenly (abrupt accretion of the shock front) or can be created and expand quickly through the flow. If the parameter exceeds the shock-existence region from both sides, the hysteresis cycle in the shock location appears.

The changes of the flow in response to the different angular momentum of the gas lead to the variable accretion rate onto the black hole. Depending on the value and on the rate of change of λ we can observe oscillatory behaviour with changing frequency, longer dips and peaks connected with the time scale of the oscillations of λ and sharp peaks determined by the minimal stable shock position.

These features can be connected with some of the variability seen in microquasars and also some low luminous AGNs like Sgr A* in the center of our Galaxy. This conclusion is strengthen by our study of non-linear features found in the lightcurves of several microquasars (Suková et al., 2016), detailed study of the microquasar XTE J1550-564 discusses also the possible outburst scenario (Suková and Janiuk, 2016). However, our method can-

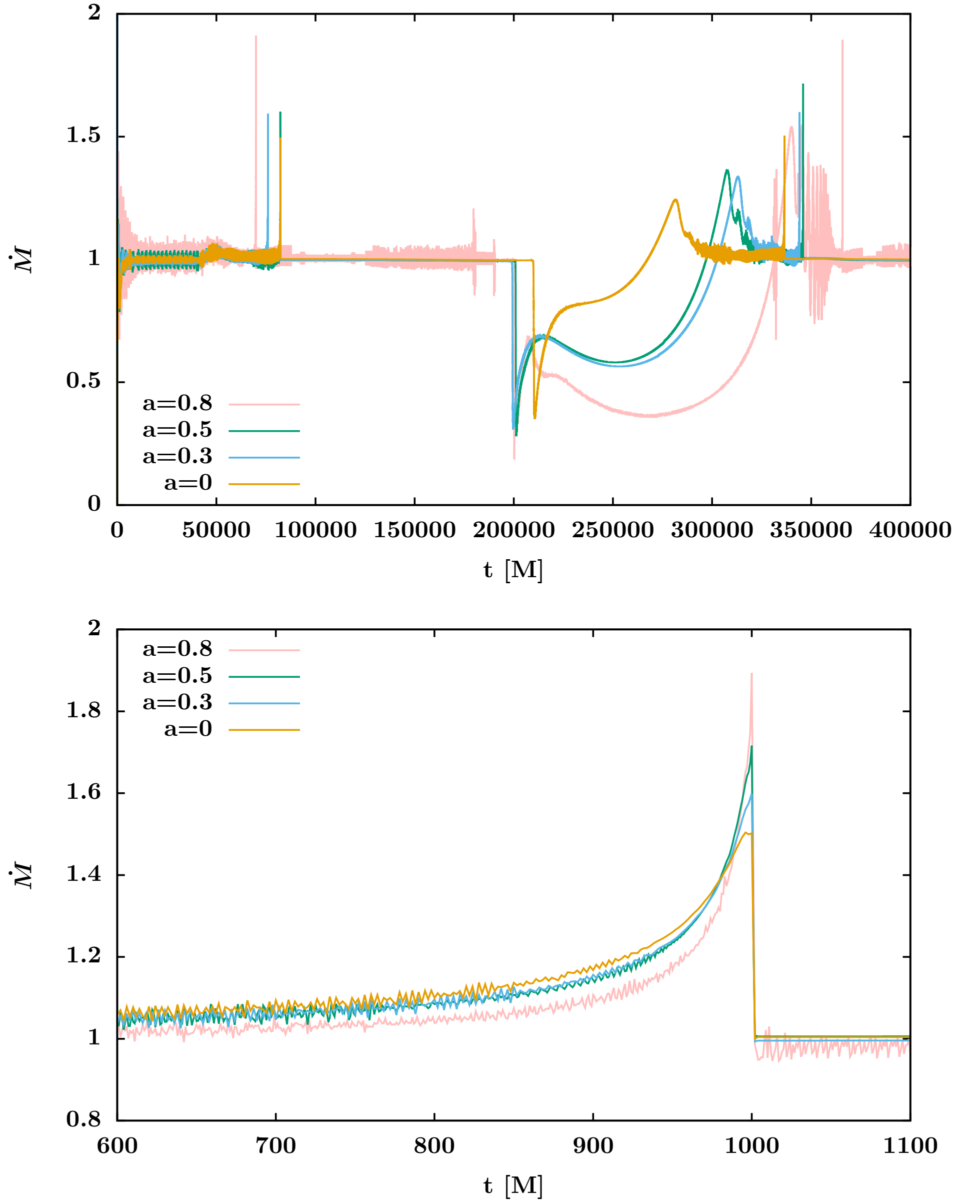

Figure 3. Time dependence of the mass accretion rate through the inner boundary for simulations with changing spin. Parameters of the runs are described in the text.

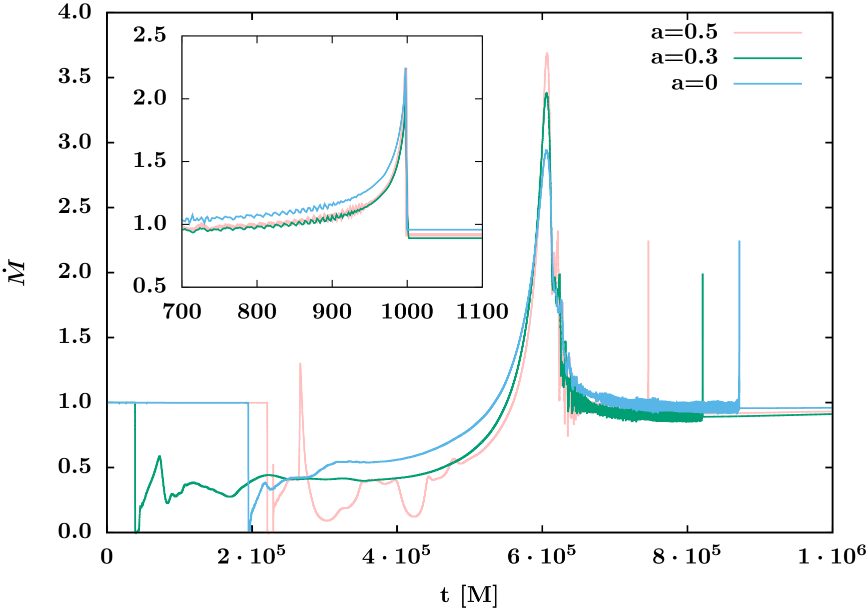

Figure 4. Time dependence of the mass accretion rate through the inner boundary for simulations with changing spin and lower energy $\epsilon = 0.0005$. The time is adjusted so that the broad peaks of all runs coincide. The inset plot shows the sharp peaks, which are aligned to $t = 1000$M. Parameters of the runs are described in the text.

not distinguish between different non-linear mechanisms of the emission and thus it cannot rule out the other possible scenarios.

Hence, more elaborate models, which include also the radiation transfer and possibly also other physical processes (e.g. the magnetic field, the two temperature accretion flows, etc.), are needed to study the observational consequences of this phenomenon, which can be directly compared to the experimental data.

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