

Levitating atmospheres around naked singularities

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ABSTRACT

For a wide class of spherically symmetric naked singularities there is a sphere within which gravity is effectively repulsive. In such spacetimes accreting matter cannot reach the singularity and will instead form a levitating atmosphere, which is kept suspended by gravity alone. The density of the atmosphere has a maximum at a definite radius. In its qualitative properties the atmosphere is analogous to the recently discussed atmospheres that are supported by radiation pressure above luminous neutron stars, however for the levitating atmospheres around a naked singularity no radiation needs to be present.

Keywords: Naked singularities – levitating atmospheres – hydrostatic equilibrium

1 INTRODUCTION

Stellar atmospheres are widely studied in the Newtonian gravity context, a general qualitative property being that their density and pressure profiles always decrease with radius. Recently, it was discovered that near-Eddington luminous neutron stars may have atmospheres detached from their surfaces if general relativistic effects are taken into account, either in the optically thin or in the optically thick case (Wielgus et al., 2015, 2016). These equilibrium configurations were called “levitating atmospheres” (Wielgus et al., 2015), since they lie at a finite distance from the stellar surface and have definite inner and outer radii. The oscillation modes of these atmospheres were studied in Abarca and Kluźniak (2016); Bollimpalli and Kluźniak (2017); Bollimpalli et al. (2019).

Here we investigate whether similar structures can appear in the absence of radiation. We find that indeed they can, if we consider spherically symmetric spacetimes generated by a central naked singularity. These naked-singularity spacetimes appear not only in general relativity, but also in solutions to modified theories of gravity. They generally present a zero-gravity radius, a stable equilibrium point for radial particle motion. Below we present equilibrium atmospheric solutions which “levitate” around these singularities, being supported solely by gravity – the central repulsive-gravity region.

2 REPULSIVE GRAVITY IN NAKED-SINGULARITY SPACETIMES

Let us consider a spherically symmetric spacetime metric of the form¹

$$ds^2 = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (1)$$

The four-acceleration of a static observer is given by $a_\mu = \partial_\mu \Phi$ (Semerák et al., 1999). For metric (1) we have only a radial component, $a_r = \Phi'(r)$. Therefore, if there is a radius r_o such that $\Phi'(r_o) = 0$, particles at rest in the sphere labelled by r_o will remain at rest. We will have then an equilibrium radius for test-particle motion. If $\Phi''(r_o) > 0$ this equilibrium is stable; moreover, for $r < r_o$ we will have a region where gravity behaves as a “repulsive force”. We call the equilibrium sphere of radius r_o the “zero-gravity sphere”. Many spherically symmetric naked-singularity spacetimes behave this way, having an equilibrium radius for particle motion at a finite distance from the singularity (e.g., Pugliese et al. (2011); Stuchlík and Schee (2014); Vieira et al. (2014); Goluchová et al. (2015); Boshkayev et al. (2016)). We will see that, whenever this is the case, a spherical shell of matter will inevitably form around this radius, whose thickness becomes larger as its peak density grows.

3 LEVITATING ATMOSPHERES

If $\Phi''(r_o) > 0$ at the zero-gravity sphere, then we expect that accreting matter onto the singularity starts accumulating around that radius, giving rise to a dense structure with a peak at r_o . We call it a levitating atmosphere, in analogy with the recently found levitating atmospheres around luminous neutron stars in general relativity (Wielgus et al., 2015).

Consider a test-fluid atmosphere around the singularity. It is described by a perfect-fluid energy-momentum tensor

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu + p g^{\mu\nu}, \quad (2)$$

where ε is the fluid’s energy density and p its pressure. Let us assume that the naked-singularity spacetime is a solution of a theory of gravity where the usual conservation laws are valid for matter. The equation of hydrostatic equilibrium is then obtained from $T^{\mu\nu}{}_{;\nu} = 0$ and, neglecting relativistic contributions to internal energy ($\varepsilon = \rho$, the mass density) and assuming $p \ll \rho$, reads (Schutz, 2009)

$$\frac{1}{\rho} \frac{dp}{dr} = -\frac{d\Phi}{dr}. \quad (3)$$

This equation has the same form of the Newtonian equation of hydrostatic equilibrium, where Φ is the gravitational potential.

¹ We work in geometrized units $c = G = 1$.

3.1 Geometrically thin approximation

By the preceding discussion, the atmospheres should have a peak of density and pressure at the zero-gravity radius r_o . If the atmospheres are geometrically thin, we can Taylor expand the function Φ to second order in $(r - r_o)$ as

$$\Phi(r) - \Phi(r_o) \approx \frac{1}{2} \Phi''(r_o) (r - r_o)^2 \quad (4)$$

and then solve the hydrostatic equilibrium equation (3) given the atmosphere's equation of state.

3.1.1 Isothermal solution

For an isothermal atmosphere with an ideal gas equation of state

$$p = [k_B T / (\mu m_p)] \rho, \quad (5)$$

where k_B is Boltzmann's constant, m_p is the proton mass, and $\mu = 1$ is the mean molecular weight for Hydrogen (since the gas is not ionized), we obtain a Gaussian profile for its pressure, peaked at $r = r_o$,

$$p(r) = p_o \exp \left[-\frac{\mu m_p}{2k_B T} \Phi''(r_o) (r - r_o)^2 \right], \quad (6)$$

with the corresponding density profile being given by the ideal gas law.

3.1.2 Polytropic solutions

For a polytropic equation of state of the form

$$p = K \rho^\gamma \quad (7)$$

we have

$$p(r) = p_o \left\{ 1 - \frac{1 - 1/\gamma}{2K^{1/\gamma} (p_o)^{1-1/\gamma}} \Phi''(r_o) (r - r_o)^2 \right\}^{\gamma/(\gamma-1)}, \quad (8)$$

with $p_o = p(r_o)$, and a corresponding density profile

$$\rho(r) = \rho_o \left\{ 1 - \frac{1 - 1/\gamma}{2K \rho_o^{(\gamma-1)}} \Phi''(r_o) (r - r_o)^2 \right\}^{1/(\gamma-1)}. \quad (9)$$

The temperature profile is given by

$$T = T_o \left\{ 1 - \left(\frac{1 - 1/\gamma}{k_B T_o / (\mu m_p)} \right) \cdot \frac{1}{2} \Phi''(r_o) (r - r_o)^2 \right\}^\gamma, \quad (10)$$

assuming an ideal gas law. We remark that the isothermal solution falls exponentially with coordinate distance from r_o in this approximation. However, we do not expect the

atmosphere to be isothermal near the singularity, so there should be a cutoff radius at a finite value of r . On the other hand, all the polytropic solutions have (positive) inner and outer edges given by the condition $p(r) = 0$.

Therefore, in the geometrically thin approximation all the profiles depend on the space-time metric only via two numbers: the zero-gravity radius r_o and the (positive) value of $\Phi''(r_o)$. In particular, the atmospheres will have the same shape regardless of the peculiarities of each spacetime; for a given equation of state, the difference will appear in the position of its peak and in the width of the profiles.

4 CONCLUSIONS

If accreting matter gradually falls onto the singularity, for instance via an accretion disc, then it will lose energy while falling into the potential well of Φ . It is a property of thin accretion discs in the presence of central repulsive gravity that their inner edge is precisely at the zero-gravity radius (e.g., [Stuchlík and Schee \(2014\)](#); [Vieira et al. \(2014\)](#)). When matter flows through this inner edge of the disk, it will spread in the polar directions over a spherical shell of the same radius. If it starts falling onto the singularity, it will have to climb the potential well generated by the central repulsive gravity region. Since the energy of the flowing matter will not be sufficient to do so, it will be driven back to larger radii and oscillate around r_o . Eventually, due to viscous forces in the fluid, it will settle down at r_o and form the levitating atmosphere.

In this way, matter will never reach the singularity. If enough matter is deposited in the atmosphere, so that the geometrically thin approximation is not valid anymore, exact solutions may give us the optical depth of denser atmospheres and therefore define whether these may be optically thick. In that case, for external observers, the singularity may appear not so different from a gas planet. Therefore a natural astrophysical process, namely accretion, may “cloak” the singularity with a dense spherical layer of optically thick gas. The question of whether this “levitating cloak” also occurs in nonspherical configurations, such as rotating singularities, deserves further investigation.

ACKNOWLEDGEMENTS

The authors thank Maciek Wielgus for interesting discussions. WK acknowledges the hospitality at Federal University of ABC, Brazil, where part of this work was developed. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001, via the Brazilian CAPES - PrInt internationalization program, and by the Polish NCN grant No. 2019/33/B/ST9/01564.

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